Exercise sheet 7
Theoretical Physics 6a (QFT): WS 2017-2018
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Exercise 1. (40 points) : Wick's Theorem

In the proof of Wick's theorem, we need the evaluation of the following commutator:

$$\left[\phi_1^+, N(\phi_2\phi_3)\right]_- = N(\underbrace{\phi_1\phi_2}\phi_3) + N(\phi_2\underbrace{\phi_1\phi_3}). \tag{1}$$

where we have defined $\phi_i \equiv \phi(x_i)$, and where ϕ_i^+ stands for the positive frequency part (annihilation part) in the field ϕ_i. Prove the above identity. *Hint:* Start by explicitly splitting the fields ϕ_2 and ϕ_3 into positive and negative frequency parts.

Exercise 2. (60 points) : Scalar $2 \to 4$ scattering

Considering the interaction Lagrangian for scalar fields

$$\mathcal{L}_1 = -\frac{\lambda}{4!}\phi^4, \tag{2}$$

and the Dyson Expansion of S-Matrix:

$$S = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int d^4x_1 \cdots \int d^4x_n T\left\{\mathcal{H}_1(x_1)\cdots\mathcal{H}_1(x_n)\right\}. \tag{3}$$

Calculate the second order ($n = 2$) S-matrix element for a process of 2 initial

bosons (of momenta p_1 and p_2) going to 4 final ones (of momenta p_3, p_4, p_5 and p_6) and draw the diagrams which arise from it (at least 2 re-orderings of the external fields).