

Exercise sheet 6  
Theoretical Physics 6a (QFT): WS 2017-2018  
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**Exercise 1. (60 points) : Scalar Quantum Electrodynamics**

**(a)(30 points)** Consider the Lagrangian for the charged Klein-Gordon Field

$$\mathcal{L} = (\partial_\mu \phi)^\dagger (\partial^\mu \phi) - m^2 \phi^\dagger \phi \quad (1)$$

- Check that this Lagrangian is not invariant under U(1) gauge transformation of the form  $\phi(x) \rightarrow \phi'(x) = e^{i\alpha(x)} \phi(x)$ ;
- Replace the derivatives by the covariant one  $D_\mu = \partial_\mu + ieA_\mu$  and check whether now the Lagrangian is invariant under the local symmetry;
- Write down the interaction Lagrangian and identify the terms. Sketch each one as a different diagrams;

**(b)(30 points)** For the free charged KG-Lagrangian in equation (1), calculate the conserved current and the conserved charge for a Lorentz boost along the x-axis.

*Hint:* Use the Lorentz transformation with generator derived in the previous exercise sheet.

## Exercise 2. (40 points) : Gauge transformation

Consider a state  $|\Psi_T\rangle$  which only contains transverse photons. Furthermore, construct a state  $|\Psi'_T\rangle$  as:

$$|\Psi'_T\rangle = \left\{ 1 + \alpha \left[ a^\dagger(\vec{k}, 3) - a^\dagger(\vec{k}, 0) \right] \right\} |\Psi_T\rangle,$$

with  $\alpha$  a constant. Show that replacing  $|\Psi_T\rangle$  by  $|\Psi'_T\rangle$  corresponds to a gauge transformation:

$$\langle \Psi'_T | A^\mu(x) | \Psi'_T \rangle = \langle \Psi_T | A^\mu(x) + \partial^\mu \Lambda | \Psi_T \rangle,$$

where  $\Lambda$  is given by:

$$\Lambda(x) = \text{Re} \left( i\alpha \frac{\sqrt{2}}{\omega_k^{3/2}} e^{-ik \cdot x} \right).$$