Exercise sheet 5
Theoretical Physics 6a (QFT): WS 2017-2018
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Exercise 1. (60 points) : The angular momentum operator

(a)(20 points) Starting from the transformation law for the classical Dirac field under Lorentz transformations show that the generators of these transformations are given by

$$M_{\mu\nu} = i(x_\mu \partial_\nu - x_\nu \partial_\mu) + \frac{1}{2}\sigma_{\mu\nu}$$

(b)(20 points) The angular momentum of the Dirac field is

$$M_{\mu\nu} = \int d^3x\, \psi^\dagger(x) \left[i(x_\mu \partial_\nu - x_\nu \partial_\mu) + \frac{1}{2}\sigma_{\mu\nu} \right] \psi(x)$$

Prove that

$$[M_{\mu\nu}, \psi(x)] = -i(x_\mu \partial_\nu - x_\nu \partial_\mu)\psi(x) - \frac{1}{2}\sigma_{\mu\nu}\psi(x)$$

(c)(20 points) Insert the normal mode expansions and construct explicitly the angular momentum tensor in terms of particles and anti-particles.

Exercise 2. (20 points) : Dirac Propagator

Derive the Feynman propagator for the Dirac Field through Green's Function. Write the final expression in the configuration space and compare it with Klein-Gordon propagator.
Hint: The propagator should satisfies the following equation

$$\left(i\partial\!\!\!/ - m\right) S_F(x-y) = i\delta^{(4)}(x-y).$$

Solve it for the propagator in the momentum space first, then apply the Fourier transformation and integrate over dp^0, considering the same $i\epsilon$ prescription as was done previously in the lectures for the Klein-Gordon field.

Exercise 3. (20 points) : Positronium

Positronium is a bound state of an e^- and a e^+. Their spins can combine into either total spin $S = 0$ or 1.

(a)(5 points) Show that the corresponding spin wavefunctions are either odd ($S = 0$) or even ($S = 1$) under exchange of the spins.

(b)(5 points) Write down all allowed total angular momentum values for a positronium state which has orbital angular momentum $L = 0$ or $L = 1$.

(c)(5 points) Show that for a state of total spin S and orbital angular momentum L, the parity P of the state (eigenvalue of the P operation) is given by $P = (-1)^{L+1}$.

(d)(5 points) Show that for a state of total spin S and orbital angular momentum L, the C-parity C of the state (eigenvalue of the charge conjugation C operation) is given by $C = (-1)^{L+S}$.