Exercise sheet 11
Theoretical Physics 3 : QM SS2017
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Exercise 1. (40 points)

In this first exercise we will practice how to couple two angular momenta j_1 and j_2, using the Clebsch-Gordan Table.

a) *(25 p.)* Write down all the possible states $|JM\rangle$ in the basis $|j_1 m_1\rangle\,|j_2 m_2\rangle$ for the compositions $\frac{1}{2} \otimes 1$ and $1 \otimes 1$ (the symbol $\otimes$ stands for the coupling of two angular momenta).

b) *(15 p.)* Check explicitly that the decompositions of the state $|\frac{5}{2}, +\frac{1}{2}\rangle$ in the basis $|\frac{1}{2} m_1\rangle\,|1 m_2\rangle\,|1 m_3\rangle$ obtained from $(\frac{1}{2} \otimes 1) \otimes 1$ and $\frac{1}{2} \otimes (1 \otimes 1)$ are the same.

Exercise 2. (30 points)

Consider a general spin-1/2 state

$$\chi = \begin{pmatrix} a \\ b \end{pmatrix}$$

which is normalized $|a|^2 + |b|^2 = 1$.

a) *(10 p.)* Show that there always exists a direction in space $\vec{n} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ such that χ is the eigenstate of the spin component along this direction $S_{\vec{n}} = \vec{n} \cdot \vec{S}$ with eigenvalue $\hbar/2$.

b) *(15 p.)* Write θ and ϕ in terms of a and b.

c) *(5 p.)* Would an analogous result hold for higher spin states?

Hint: Count the number of degrees of freedom.

Exercise 3. (30 points)

a) *(15 p.)* Derive the spin matrices S_x, S_y, S_z in the basis $|s, s_z\rangle$ for $s = 1$.

b) *(15 p.)* Find the eigenvalues and the normalized eigenvectors of S_x and S_y in that basis.

Hint: The general relation $S_\pm|s, s_z\rangle = \hbar\sqrt{s(s+1) - s_z(s_z \pm 1)}|s, s_z \pm 1\rangle$ can we useful.