

# SCHRÖDINGER'S CAT & KITTENS

- 1) QUASI-CLASSICAL STATES OF HARMONIC OSCILLATOR (H.O.)
- 2) SCHRÖDINGER - CAT STATE
- 3) FRAGILITY OF QUANTUM SUPERPOSITION



# ⇒ 1) QUASI-CLASSICAL STATES OF H.O.

↳ H.O. OPERATOR METHOD

$$\bullet \hat{H} = \frac{\hat{P}^2}{2m} + \frac{1}{2} m \omega^2 \hat{X}^2$$

$$\downarrow \left\{ \begin{array}{l} \hat{X} \equiv \sqrt{\frac{m\omega}{\hbar}} \hat{x} \\ \hat{P} \equiv \frac{1}{\sqrt{m\hbar\omega}} \hat{p} \end{array} \right.$$

$$\begin{aligned} \hat{H} &= \frac{\hbar\omega}{2} (\hat{P}^2 + \hat{X}^2) \\ &= \frac{\hbar\omega}{2} (\hat{X}^2 - (i\hat{P})^2) \end{aligned}$$

$$\left\{ \begin{array}{l} \hat{a} \equiv \frac{1}{\sqrt{2}} (\hat{X} + i\hat{P}) \\ \hat{a}^{\dagger} \equiv \frac{1}{\sqrt{2}} (\hat{X} - i\hat{P}) \end{array} \right.$$

IN CHAPTER 2 WE DENOTED

$$\hat{a}_- = \hat{a}$$

$$\hat{a}_+ = \hat{a}_-^{\dagger} = \hat{a}^{\dagger}$$

$$[\hat{a}, \hat{a}^{\dagger}] = 1.$$

• SOLUTIONS

$$\hat{H} |m\rangle = E_m |m\rangle$$

$$\psi_m(x) \equiv \langle x | m \rangle$$

$$\left\{ \begin{array}{l} \hat{a} |m\rangle = \sqrt{m} |m-1\rangle \\ \hat{a}^+ |m\rangle = \sqrt{m+1} |m+1\rangle \end{array} \right.$$

$$\hat{a}^+ \hat{a} |m\rangle = m |m\rangle \quad m = 0, 1, 2, \dots$$

$$\hat{H} = \hbar\omega \left( \hat{a}^+ \hat{a} + \frac{1}{2} \right)$$

$$E_m = \hbar\omega \left( m + \frac{1}{2} \right) \quad m = 0, 1, 2, \dots$$

• GROUND STATE  $m=0$

$$\hat{a} |0\rangle = 0$$

$$(\hat{X} + i\hat{P}) |0\rangle = 0$$

$$\langle x | \hat{X} + i\hat{P} |0\rangle = 0$$

↳ IN COORDINATE BASIS

$$\left( \sqrt{\frac{m\omega}{\hbar}} \left( x + \sqrt{\frac{\hbar}{m\omega}} \frac{d}{dx} \right) \right) \psi_0(x) = 0$$

$$\psi_0(x) = C e^{-\frac{m\omega}{2\hbar} x^2} \quad \left( C = \left( \frac{m\omega}{\pi\hbar} \right)^{1/4} \right)$$

## QUASI-CLASSICAL STATE

- EIGENSTATE OF  $\hat{a}$  WITH EIGENVALUE  $\alpha$  (COMPLEX)

$$\underline{\underline{\hat{a} |\alpha\rangle = \alpha |\alpha\rangle}} \quad \langle \alpha | \hat{a}^\dagger = \langle \alpha | \alpha^*$$

$\leadsto$   $|m\rangle$  FORM A COMPLETE SET

$$|\alpha\rangle = \sum_{m=0}^{\infty} \alpha_m |m\rangle$$

$$\alpha_m = \langle m | \alpha \rangle$$

$$\leadsto \hat{a} |\alpha\rangle = \sum_{m=1}^{\infty} \alpha_m \sqrt{m} |m-1\rangle$$

$$= \sum_{m=0}^{\infty} \alpha_{m+1} \sqrt{m+1} |m\rangle$$

$$\alpha |\alpha\rangle = \sum_{m=0}^{\infty} \alpha \alpha_m |m\rangle$$

$\Downarrow$

$$\alpha_{m+1} = \frac{1}{\sqrt{m+1}} \alpha \alpha_m$$

$\leadsto$  SUPPOSE WE CHOOSE  $\alpha_0 = C$  (CONSTANT)

RECURSIVELY

$$\alpha_1 = \frac{\alpha}{\sqrt{1}} \cdot C$$

$$\alpha_2 = \frac{\alpha^2}{\sqrt{2!}} C$$

$\vdots$

$$\alpha_m = \frac{\alpha^m}{\sqrt{m!}} c$$

$$|\alpha\rangle = c \sum_{m=0}^{\infty} \frac{\alpha^m}{\sqrt{m!}} |m\rangle$$

→ NORMALIZATION  $\langle \alpha | \alpha \rangle = 1$

$$1 = |c|^2 \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{\alpha^{*n} \alpha^m}{\sqrt{n! m!}} \underbrace{\langle n | m \rangle}_{\delta_{nm}}$$

$$1 = |c|^2 \sum_{m=0}^{\infty} \frac{|\alpha|^{2m}}{m!} = |c|^2 e^{|\alpha|^2}$$

$$|c| = e^{-|\alpha|^2/2}$$

$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{m=0}^{\infty} \frac{\alpha^m}{\sqrt{m!}} |m\rangle$$

(ALSO CALLED A COHERENT STATE)

$$\hookrightarrow \langle \hat{H} \rangle, \langle \hat{X} \rangle, \langle \hat{P} \rangle, \sigma_x^2, \sigma_p^2 \text{ in } |\alpha\rangle$$

$$\begin{aligned} \bullet \langle \alpha | \hat{H} | \alpha \rangle &= \hbar\omega \langle \alpha | \hat{a}^\dagger \hat{a} + \frac{1}{2} | \alpha \rangle \\ &= \hbar\omega (|\alpha|^2 + \frac{1}{2}) \end{aligned}$$

$|\alpha|^2$  CAN BE INTERPRETED AS  
AVERAGE VALUE OF EXCITATION (LEVELS  
ENERGY)

$$\begin{aligned} \bullet \langle \hat{X} \rangle &= \sqrt{\frac{\hbar}{m\omega}} \langle \alpha | \hat{X} | \alpha \rangle \\ &= \sqrt{\frac{\hbar}{m\omega}} \frac{1}{\sqrt{2}} \langle \alpha | \hat{a} + \hat{a}^\dagger | \alpha \rangle \\ &= \sqrt{\frac{\hbar}{m\omega}} \frac{1}{\sqrt{2}} \{ \alpha + \alpha^* \} = \sqrt{\frac{\hbar}{m\omega}} \sqrt{2} \operatorname{Re} \alpha \end{aligned}$$

$$\begin{aligned} \bullet \langle \hat{P} \rangle &= \sqrt{m\hbar\omega} \langle \alpha | \hat{P} | \alpha \rangle \\ &= \sqrt{m\hbar\omega} \frac{1}{2i} \frac{1}{\sqrt{2}} \langle \alpha | \hat{a} - \hat{a}^\dagger | \alpha \rangle \\ &= \sqrt{m\hbar\omega} \frac{1}{\sqrt{2}} \frac{1}{i} \{ \alpha - \alpha^* \} \\ &= \sqrt{m\hbar\omega} \sqrt{2} \operatorname{Im} \alpha \end{aligned}$$

$$\begin{aligned} \bullet \text{ FOR SCALED VARIABLES } \quad \langle \hat{X} \rangle &= \sqrt{2} \operatorname{Re} \alpha \\ \langle \hat{P} \rangle &= \sqrt{2} \operatorname{Im} \alpha \end{aligned}$$

$$\begin{aligned}
 \sigma_x^2 &= \langle \alpha | (\hat{x} - \langle \hat{x} \rangle)^2 | \alpha \rangle \\
 &= \langle \alpha | \hat{x}^2 | \alpha \rangle - \langle \hat{x} \rangle^2 \\
 &= \frac{\hbar}{m\omega} \frac{1}{2} \langle \alpha | (\hat{a} + \hat{a}^\dagger)(\hat{a} + \hat{a}^\dagger) | \alpha \rangle - \langle \hat{x} \rangle^2 \\
 &= \frac{\hbar}{m\omega} \frac{1}{2} \langle \alpha | \hat{a}^2 + \hat{a}^{\dagger 2} + \underbrace{\hat{a}\hat{a}^\dagger + \hat{a}^\dagger\hat{a}}_{1 + \hat{a}^\dagger\hat{a}} | \alpha \rangle \\
 &\quad - \langle \hat{x} \rangle^2 \\
 &= \frac{\hbar}{m\omega} \frac{1}{2} \left( \alpha^2 + \alpha^{*2} + 1 + 2\alpha\alpha^* \right) \\
 &\quad - \langle \hat{x} \rangle^2 \\
 &= \frac{\hbar}{m\omega} \frac{1}{2} \left( (\alpha + \alpha^*)^2 + 1 \right) - \frac{\hbar}{m\omega} \frac{1}{2} (\alpha + \alpha^*)^2
 \end{aligned}$$

$$\boxed{\sigma_x^2 = \frac{\hbar}{2m\omega}} \quad \text{INDEPENDENT OF } \alpha \quad \left( \sigma_x = \frac{1}{\sqrt{2}} \right)$$

$$\begin{aligned}
 \sigma_p^2 &= \langle \alpha | (\hat{p} - \langle \hat{p} \rangle)^2 | \alpha \rangle \\
 &= \hbar m\omega \frac{1}{2} \left( \left( \frac{\alpha - \alpha^*}{i} \right)^2 + 1 \right) - \hbar m\omega \frac{1}{2} \left( \frac{\alpha - \alpha^*}{i} \right)^2
 \end{aligned}$$

$$\boxed{\sigma_p^2 = \frac{\hbar m\omega}{2}} \quad \left( \sigma_p = \frac{1}{\sqrt{2}} \right)$$

$$\sigma_x \sigma_p = \frac{\hbar}{2} \quad \text{MINIMUM UNCERTAINTY STATE!}$$

↳ WAVEFUNCTION IN COORDINATE SPACE

$$\Psi_{\alpha}(X) \equiv \langle X | \alpha \rangle$$

$$\hat{a} | \alpha \rangle = \alpha | \alpha \rangle$$

$$\langle X | \hat{a} | \alpha \rangle = \alpha \langle X | \alpha \rangle$$

$$\hat{a} = \frac{1}{\sqrt{2}} \left( \hat{X} + i \hat{P} \right) \quad \Rightarrow \quad \hat{P} = -i \frac{d}{dX}$$

$$\frac{1}{\sqrt{2}} \left( X + \frac{d}{dX} \right) \Psi_{\alpha}(X) = \alpha \Psi_{\alpha}(X)$$

$$\frac{d\Psi_{\alpha}}{dX} = -(\sqrt{2}\alpha + X) \Psi_{\alpha}(X)$$

$$\downarrow$$

$$\underline{\underline{\Psi_{\alpha}(X) = C e^{-\frac{1}{2}(X - \sqrt{2}\alpha)^2}}}$$

NOTE  $\langle \hat{X} \rangle = \sqrt{2} \operatorname{Re} \alpha$

C is NORMALIZATION CONSTANT



## ↳ WAVE FUNCTION IN MOMENTUM SPACE

$$\varphi_\alpha(P) \equiv \langle P | \alpha \rangle$$

$$\langle P | \hat{a} | \alpha \rangle = \alpha \langle P | \alpha \rangle$$

$$\langle P | \frac{1}{\sqrt{2}} (\hat{X} + i\hat{P}) | \alpha \rangle = \alpha \langle P | \alpha \rangle$$

$$\downarrow \quad \hat{X} = i \frac{d}{dP}$$

$$\frac{i}{\sqrt{2}} \left( \frac{d\varphi_\alpha}{dP} + P \varphi_\alpha(P) \right) = \alpha \varphi_\alpha(P)$$

$$\frac{d\varphi_\alpha}{dP} = - (P + i\sqrt{2}\alpha) \varphi_\alpha(P)$$

$$\varphi_\alpha(P) = C' e^{-\frac{1}{2}(P + i\sqrt{2}\alpha)^2}$$


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$$\langle \hat{P} \rangle = \sqrt{2} \operatorname{Im} \alpha$$

## ↳ TIME EVOLUTION OF QUASI-CLASSICAL STATE

- SUPPOSE AT  $t=0$ , OSCILLATOR IS IN QUASI-CLASSICAL STATE

$$|\psi(t=0)\rangle = |\alpha_0\rangle \quad \text{WITH } \alpha_0 = \rho e^{i\phi}$$

$\rho, \phi \in \mathbb{R}$   
 $\rho > 0$

- TIME EVOLUTION

$$|\psi(t)\rangle = e^{-\frac{|\alpha_0|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha_0^n}{n!} e^{-\frac{i}{\hbar} E_n t} |n\rangle$$

SATISFIES  $\hat{H} |\psi(t)\rangle = i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle$

$$E_n = \hbar\omega(n + \frac{1}{2}) \quad \downarrow$$

$$|\psi(t)\rangle = e^{-\frac{|\alpha_0|^2}{2}} e^{-i\omega t/2} \sum_{n=0}^{\infty} \frac{(\alpha_0 e^{-i\omega t})^n}{n!} |n\rangle$$

$$\alpha(t) \equiv \alpha_0 e^{-i\omega t} = \rho e^{-i(\omega t - \phi)}$$


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NOTE  $|\alpha(t)| = |\alpha_0| = \rho$

$$|\psi(t)\rangle = e^{-i\omega t/2} |\alpha(t)\rangle$$


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AT LATER TIME  $t$ , OSCILLATOR IS STILL IN A QUASI-CLASSICAL STATE  $\alpha(t)$

$$\begin{aligned}
 \bullet \quad \langle x \rangle_t &= \langle \psi(t) | \hat{x} | \psi(t) \rangle \\
 &= \langle \alpha(t) | \hat{x} | \alpha(t) \rangle \\
 &= \sqrt{\frac{2\hbar}{m\omega}} \operatorname{Re} \alpha(t) \\
 &= \sqrt{\frac{2\hbar}{m\omega}} \rho \cos(\omega t - \phi) = x_0 \cos(\omega t - \phi)
 \end{aligned}$$

$$\begin{aligned}
 \bullet \quad \langle p \rangle_t &= \langle \psi(t) | \hat{p} | \psi(t) \rangle \\
 &= \langle \alpha(t) | \hat{p} | \alpha(t) \rangle \\
 &= \sqrt{2m\hbar\omega} \operatorname{Im} \alpha(t) \\
 &= -\sqrt{2m\hbar\omega} \rho \sin(\omega t - \phi) \\
 &= -p_0 \sin(\omega t - \phi)
 \end{aligned}$$

NOTE  $\langle p \rangle_t = m \frac{d}{dt} \langle x \rangle_t$

$\langle x \rangle_t$ ,  $\langle p \rangle_t$  SOLUTIONS OF CLASSICAL OPERATOR !

NOTE  $\frac{\sigma_x}{|\langle x \rangle_t|} \approx \frac{1}{2\rho} \ll 1$

$\frac{\sigma_p}{|\langle p \rangle_t|} \approx \frac{1}{2\rho} \ll 1$

FOR  $|\alpha| \gg 1$

BOTH POSITION & MOMENTUM QUITE ACCURATELY DEFINED

↳ EXAMPLE PENDULUM  $l = 1 \text{ m}$   
 $m = 1 \text{ g}$

DESCRIBED BY QUASI-CLASSICAL STATE

$$t=0: \quad x_0 = 10^{-6} \text{ m}$$

$$\omega = \sqrt{\frac{g}{l}} = 3.13 \text{ s}^{-1}$$

$$\alpha(t=0) = \alpha_0 = \rho = x_0 \sqrt{\frac{m\omega}{2\hbar}}$$

$$\approx 10^{-6} \left( \frac{10^{-3} \cdot 3.13}{(2) 1.054 \cdot 10^{-34}} \right)^{1/2}$$

$$\underline{\underline{\alpha_0 = (3.9) \cdot 10^9}}$$

$$\frac{\Delta \sigma_x}{|\langle x \rangle_t|} \approx 10^{-10}$$

$$\frac{\sigma_p}{|\langle p \rangle_t|} \approx 10^{-10}$$

$$\text{AFTER } \frac{T}{4} \quad \omega \cdot \frac{T}{4} = \frac{\pi}{2}$$

$$\alpha\left(\frac{T}{4}\right) = -i \alpha_0$$

## ⇒ 2) SCHRÖDINGER - CAT STATE

- AT  $t=0$  : STATE IS IN QUASI-CLASSICAL STATE

$$|\Psi(t=0)\rangle = |\alpha\rangle$$

- BETWEEN  $[0, T]$  ONE ADDS A COUPLING

$$\hat{W} = \hbar g (\hat{a}^\dagger + \hat{a})^2$$

WITH  $g \gg \omega$  AND  $\omega T \ll 1$

⇓

HAMILTONIAN DURING  $[0, T]$  IS TO GOOD APPROXIMATION GIVEN BY  $\hat{W}$

- EIGENSTATES OF  $\hat{W}$  →  $|m\rangle$   
EIGENVALUES →  $\hbar g m^2$  }  $\hat{W}|m\rangle = \hbar g m^2 |m\rangle$

$$|\Psi(t)\rangle = e^{-|\alpha|^2/2} \sum_{m=0}^{\infty} \frac{\alpha^m}{\sqrt{m!}} e^{-i g m^2 t} |m\rangle$$

- FOR SPECIAL CASE  $T = \frac{\pi}{2g}$

$$e^{-i g m^2 T} = e^{-i \frac{\pi}{2} m^2} = \begin{cases} -i, & m \text{ ODD} \\ 1, & m \text{ EVEN} \end{cases}$$

$$= \frac{1}{\sqrt{2}} \left\{ e^{-i \frac{\pi}{4}} + e^{i \frac{\pi}{4}} (-1)^m \right\}$$

$$|\Psi(T)\rangle = \frac{1}{\sqrt{2}} \left\{ e^{-i\frac{\pi}{4}} |\alpha\rangle + e^{i\frac{\pi}{4}} |-\alpha\rangle \right\}$$

- FOR  $\alpha = i\rho$   $\rho \in \mathbb{R}$

$$\langle \hat{X} \rangle = 0$$

$$\langle \hat{P} \rangle = \pm \underbrace{\sqrt{2m\hbar\omega}}_{P_0} \rho \quad \text{FOR } |\pm\alpha\rangle$$

FOR  $|\alpha| \gg 1$   $|\alpha\rangle$  AND  $|-\alpha\rangle$

ARE MACROSCOPICALLY DIFFERENT

(e.g. 'DEAD' OR 'ALIVE' STATES OF SCHRÖDINGER'S CAT)

- PROBABILITY DISTRIBUTION FOR POSITION

$$P(X) = |\langle X | \Psi(T) \rangle|^2$$

$$= \frac{1}{2} \left| e^{-i\frac{\pi}{4}} \Psi_{\alpha}(X) + e^{i\frac{\pi}{4}} \Psi_{-\alpha}(X) \right|^2$$

$$\downarrow \Psi_{\alpha}(X) = C e^{-\frac{1}{2}(X - \sqrt{2}\alpha)^2}$$

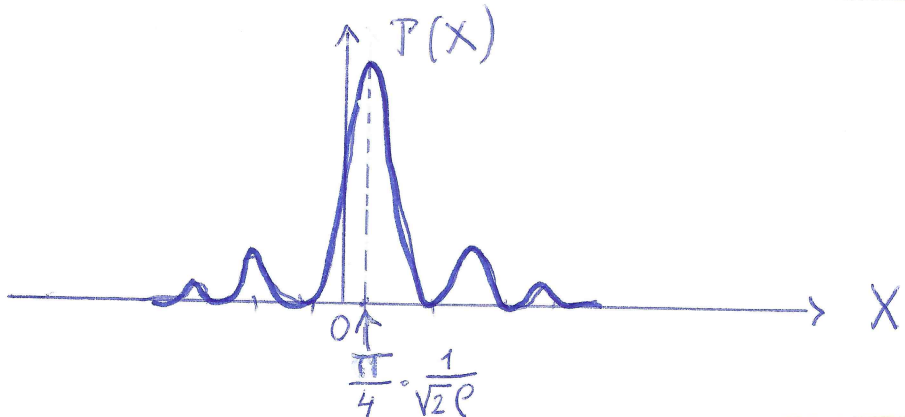
$$= |C|^2 \frac{1}{2} \left| e^{-i\frac{\pi}{4}} e^{-\frac{1}{2}(X - i\sqrt{2}\rho)^2} \right.$$

$$\left. + e^{i\frac{\pi}{4}} e^{-\frac{1}{2}(X + i\sqrt{2}\rho)^2} \right|^2$$

$$= \frac{|C|^2}{2} e^{-X^2} e^{2\rho^2} \left| e^{-i\frac{\pi}{4} + i\sqrt{2}\rho X} \right.$$

$$\left. + e^{i\frac{\pi}{4} - i\sqrt{2}\rho X} \right|^2$$

$$P(X) \approx e^{-X^2} \cos^2\left(X\rho\sqrt{2} - \frac{\pi}{4}\right)$$



• PROBABILITY DISTRIBUTION FOR MOMENTUM

$$P(P) = |\langle P | \Psi(T) \rangle|^2$$

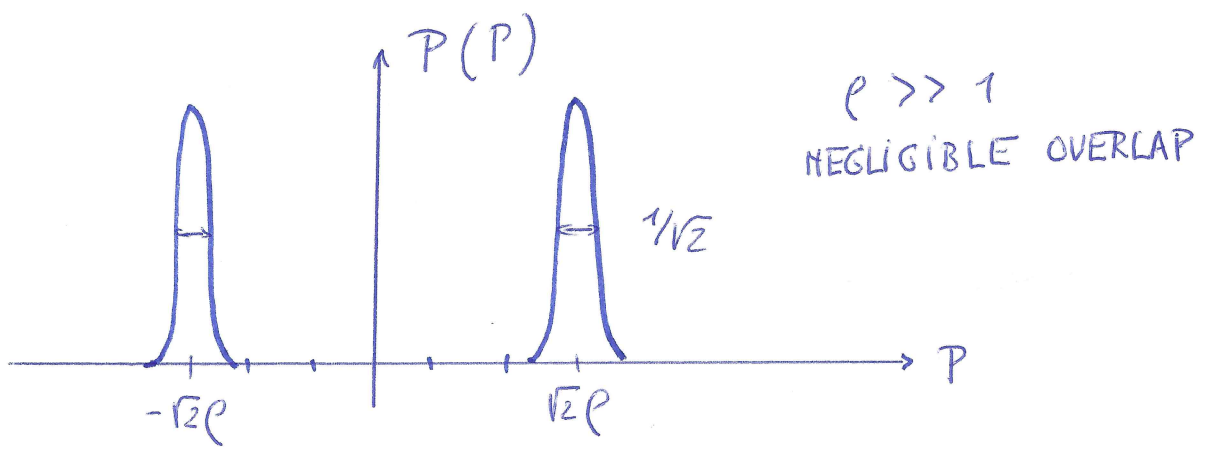
$$= \frac{1}{2} \left| e^{-i\frac{\pi}{4}} \varphi_{\alpha}(P) + e^{i\frac{\pi}{4}} \varphi_{-\alpha}(P) \right|^2$$

$$\downarrow \varphi_{\alpha}(P) = C' e^{-\frac{1}{2}(P + i\sqrt{2}\alpha)^2}$$

$$= \frac{|C'|^2}{2} \left| e^{-i\frac{\pi}{4}} e^{-\frac{1}{2}(P - \sqrt{2}\rho)^2} + e^{i\frac{\pi}{4}} e^{-\frac{1}{2}(P + \sqrt{2}\rho)^2} \right|^2$$

$$P(P) \approx e^{-\frac{1}{2}(P - \sqrt{2}\rho)^2} + e^{-\frac{1}{2}(P + \sqrt{2}\rho)^2}$$

↑ GAUSSIAN CENTERED AT  $\langle \hat{P} \rangle = \sqrt{2}\rho$       ↑ GAUSSIAN CENTERED AT  $\langle \hat{P} \rangle = -\sqrt{2}\rho$



- QM SUPERPOSITION

PREPARE  $N$  SYSTEMS IN

STATE  $|\Psi(T)\rangle$  : QM SUPERPOSITION OF 2  
QUASI-CLASSICAL STATES  
(i.e. MACROSCOPICALLY DIFFERENT)

& PERFORM MEASUREMENTS OF  $\mathcal{P}$  ON EACH SYSTEM

$$\underbrace{\frac{1}{\sqrt{2}}}_{\sigma_{\mathcal{P}}} \ll \delta \mathcal{P} \ll \underbrace{\sqrt{2} \rho}_{\langle \mathcal{P} \rangle}$$

↑  
RESOLUTION OF MEASURING APPARATUS

FOR  $N \gg 1$  :  $\sim N/2$  TIMES ONE MEASURES  $\mathcal{P} = \sqrt{2} \rho$   
 $\sim N/2$  " " "  $\mathcal{P} = -\sqrt{2} \rho$

- STATISTICAL MIXTURE

INSTEAD OF  $N$  SYSTEMS IN STATE  $|\Psi(T)\rangle$

CONSIDER STATISTICAL MIXTURE OF

$N/2$  SYSTEMS IN  $|\alpha\rangle$

$N/2$  " "  $|\neg\alpha\rangle$

& PERFORM MEASUREMENTS OF  $\mathcal{P}$  ON EACH SYSTEM

$N/2$  SYSTEMS IN  $|\alpha\rangle \rightarrow \mathcal{P} = \sqrt{2} \rho$

$N/2$  " "  $|\neg\alpha\rangle \rightarrow \mathcal{P} = -\sqrt{2} \rho$

∞ SAME RESULT IN BOTH CASES



- CAN WE DISTINGUISH BETWEEN BOTH CASES ?

OR ↗ N SYSTEMS IN QM SUPERPOSITION  $|\Psi(T)\rangle$   
 ↘ STATISTICAL MIXTURE OF N/2 SYSTEMS IN  $|\alpha\rangle$   
 N/2 " "  $|\alpha\rangle$   
 N/2 " "  $|\beta\rangle$

MEASURE POSITION IN BOTH CASES  
 ASSUMING THAT RESOLUTION

$$\delta x \ll \frac{1}{\rho} \quad (\text{i.e. ONE CAN FOLLOW OSCILLATIONS IN } \cos^2(x\rho\sqrt{2} - \pi/4))$$

↳ FOR QM SUPERPOSITION

$$\text{WE MEASURE } P(x) \sim e^{-x^2} \cos^2(x\rho\sqrt{2} - \frac{\pi}{4})$$

(OSCILLATING FUNCTION MODULATED BY GAUSSIAN)

↳ FOR STATISTICAL MIXTURE

WE MEASURE (SAME FOR  $|\alpha\rangle$  AND  $|\beta\rangle$ )

$$P(x) \sim |\Psi_{\alpha}(x)|^2 \sim e^{-x^2} \quad (\text{GAUSSIAN})$$

∴ WE CAN DISTINGUISH BOTH SITUATIONS

IF  $\delta x \ll \frac{1}{\rho}$

$$\delta x \ll \sqrt{\frac{\hbar}{m\omega}} \frac{1}{\rho}$$

FOR EXAMPLE OF PENDULUM

$$\delta x \ll \frac{x_0}{\rho^2} \sim \frac{10^{-6}}{10^{19}} \text{ m} \sim 10^{-25} \text{ m}$$

RESOLUTION NOT POSSIBLE IN PRACTICE

### ⇒ 3) FRAGILITY OF QUANTUM SUPERPOSITION

- IN REALISTIC SITUATION, OSCILLATOR IS COUPLED TO ENVIRONMENT  
 ↳ THIS WILL DETERMINE HOW LONG ONE CAN DISCRIMINATE BETWEEN QM SUPERPOSITION AND A STATISTICAL MIXTURE (HALF IN ONE STATE, HALF IN OTHER STATE)

- DISSIPATION MODEL FOR QUASI-CLASSICAL STATE

$t=0$  OSCILLATOR IS IN QUASI-CLASSICAL STATE  $|\alpha_0\rangle$   
 ENVIRONMENT  $|\chi_e(0)\rangle$

STATE OF TOTAL SYSTEM

$$|\Psi(0)\rangle = |\alpha_0\rangle |\chi_e(0)\rangle$$

$t>0$   $|\Psi(t)\rangle = |\alpha_1\rangle |\chi_e(t)\rangle$

$$\alpha_1 = \alpha(t) e^{-\gamma t} e^{-i\omega t}$$

$$\alpha(t) = \alpha_0 e^{-\gamma t}$$

$\gamma > 0$  ( $\gamma \in \mathbb{R}$ ) DAMPING OF STATE DUE TO COUPLING

- ENERGY OF OSCILLATOR AT TIME  $t$

$$\begin{aligned} \langle \alpha_1 | \hat{H} | \alpha_1 \rangle &= \hbar\omega \left( |\alpha_1|^2 + \frac{1}{2} \right) \\ &= \hbar\omega \left( |\alpha_0|^2 e^{-2\gamma t} + \frac{1}{2} \right) \end{aligned}$$

FOR  $t \gg \frac{1}{\gamma}$  → OSCILLATOR IS IN ITS GROUND STATE

FOR  $t \ll \frac{1}{2\gamma}$  → ENERGY ACQUIRED BY ENVIRONMENT  
 $E(0) - E(t) \approx \hbar\omega |\alpha_0|^2 2\gamma t$

## DISSIPATION MODEL FOR SCHRÖDINGER CAT STATE

$t=0$  : OSCILLATOR IS IN SCHRÖDINGER CAT STATE

$$\frac{1}{\sqrt{2}} \left\{ e^{-i\pi/4} |\alpha_0\rangle + e^{i\pi/4} |-\alpha_0\rangle \right\}$$

TOTAL SYSTEM

$$|\Psi(0)\rangle = \frac{1}{\sqrt{2}} \left\{ e^{-i\pi/4} |\alpha_0\rangle + e^{i\pi/4} |-\alpha_0\rangle \right\} |\chi_e(0)\rangle$$

$$\begin{aligned} \underline{t > 0} \quad |\Psi(t)\rangle = & \frac{1}{\sqrt{2}} \left\{ e^{-i\pi t/4} |\alpha_1\rangle |\chi_e^+(t)\rangle \right. \\ & \left. + e^{i\pi t/4} |-\alpha_1\rangle |\chi_e^-(t)\rangle \right\} \end{aligned}$$

$|\chi_e^+(t)\rangle, |\chi_e^-(t)\rangle$  2 NORMALIZED STATES OF ENVIRONMENT THAT ARE A PRIORI DIFFERENT (NOT ORTHOGONAL)

↳ PROBABILITY DISTRIBUTION OF OSCILLATOR'S POSITION INDEPENDENT OF STATE OF ENVIRONMENT

$$\begin{aligned} P(x) &= |\langle x | \Psi(t) \rangle|^2 \\ &= \frac{1}{2} \left\{ e^{i\pi t/4} \psi_{\alpha_1}^*(x) \langle \chi_e^+(t) | + e^{-i\pi t/4} \psi_{-\alpha_1}^*(x) \langle \chi_e^-(t) | \right\} \\ &\quad \cdot \left\{ e^{-i\pi t/4} \psi_{\alpha_1}(x) | \chi_e^+(t) \rangle + e^{i\pi t/4} \psi_{-\alpha_1}(x) | \chi_e^-(t) \rangle \right\} \end{aligned}$$

$$\begin{aligned}
 P(x) &= \frac{1}{2} \left\{ |\psi_{\alpha_1}(x)|^2 + |\psi_{-\alpha_1}(x)|^2 \right. \\
 &\quad + i \psi_{\alpha_1}^*(x) \psi_{-\alpha_1}(x) \langle \chi_e^+(t) | \chi_e^-(t) \rangle \\
 &\quad \left. - i \psi_{-\alpha_1}^*(x) \psi_{\alpha_1}(x) \langle \chi_e^-(t) | \chi_e^+(t) \rangle \right\} \\
 &= \frac{1}{2} \left\{ |\psi_{\alpha_1}(x)|^2 + |\psi_{-\alpha_1}(x)|^2 \right. \\
 &\quad \left. + 2 \operatorname{Re} \left[ i \psi_{\alpha_1}^*(x) \psi_{-\alpha_1}(x) \underbrace{\langle \chi_e^+(t) | \chi_e^-(t) \rangle}_{\eta} \right] \right\}
 \end{aligned}$$

|| SAME AS BEFORE BUT OSCILLATION  
DAMPED BY FACTOR  $\eta$

$$0 \leq \eta \leq 1$$

• PROBABILITY DISTRIBUTION FOR MOMENTUM

$$\begin{aligned}
 P(p) &= \frac{1}{2} \left\{ |\varphi_{\alpha_1}(p)|^2 + |\varphi_{-\alpha_1}(p)|^2 \right. \\
 &\quad \left. + 2 \operatorname{Re} \left[ i \varphi_{\alpha_1}^*(p) \varphi_{-\alpha_1}(p) \eta \right] \right\}
 \end{aligned}$$

FOR  $p \gg 1$  OVERLAP OF  
2 GAUSSIANS IS NEGLIGIBLE

$$\approx \frac{1}{2} \left\{ |\varphi_{\alpha_1}(p)|^2 + |\varphi_{-\alpha_1}(p)|^2 \right\}$$

- POSITION MEASUREMENT CAN DISTINGUISH BETWEEN QM SUPERPOSITION & STATISTICAL MIXTURE PROVIDED  $\eta$  IS NOT TOO SMALL e.g.  $\eta \gg \frac{1}{10}$

- ENVIRONMENT MODEL: 2<sup>o</sup> OSCILLATOR (SAME  $m, \omega$ )

$$t=0 \quad |\chi_e^+(0)\rangle = |0\rangle \quad \text{GROUND STATE}$$

$$t>0 \quad |\chi_e^\pm(t)\rangle = |\pm\beta\rangle \quad \text{QUASI-CLASSICAL STATES}$$

$$\text{AND ASSUME FOR } \gamma t \ll 1 \Rightarrow |\beta|^2 = 2\gamma t |\alpha_0|^2$$

$$\hookrightarrow \eta = \langle \chi_e^+(t) | \chi_e^-(t) \rangle$$

$$= \langle \beta | -\beta \rangle$$

$$= e^{-|\beta|^2} \sum_{m=0}^{\infty} (-1)^m \frac{|\beta|^{2m}}{m!}$$

$$= e^{-2|\beta|^2}$$

- $\hookrightarrow$  IN ORDER TO DISTINGUISH BETWEEN QM SUPERPOSITION & STATISTICAL MIXTURE

$$\eta = e^{-2|\beta|^2} \gg \frac{1}{10} \quad \Leftrightarrow |\beta| \lesssim 1$$

↳ ENERGY OF 1<sup>o</sup> OSCILLATOR FOR  $t \ll \frac{1}{\gamma}$

$$E(t) = E(0) - 2\gamma t |\alpha_0|^2 \hbar \omega$$

↳ ENERGY OF 2<sup>o</sup> OSCILLATOR

$$E'(t) = \frac{\hbar \omega}{2} + 2\gamma t |\alpha_0|^2 \hbar \omega$$

$$= \hbar \omega \left( |\beta|^2 + \frac{1}{2} \right)$$

$$\therefore |\beta| \lesssim 1 \iff 2\gamma t |\alpha_0|^2 \lesssim 1$$

IF A SINGLE ENERGY QUANTUM IS EXCHANGED  
IT IS DIFFICULT TO DISTINGUISH BETWEEN  
QM SUPERPOSITION & STATISTICAL MIXTURE

↳ FOR PENDULUM EXAMPLE  $\alpha_0 \approx 10^9$

$$(2\gamma)^{-1} = 1 \text{ year} \approx 3 \cdot 10^7 \text{ s}$$

$$|\beta| = 1 \iff t \approx 10^{+7} 10^{-18} \approx 10^{-11} \text{ s} \quad \nabla$$

↑  
LIFETIME OF  
QM SUPER-  
POSITION

EVEN WITH VERY WEAK DISSIPATION

AFTER  $10^{-11} \text{ s}$  WE CANNOT DISTINGUISH

ANY MORE BETWEEN QM SUPERPOSITION

↳ STATISTICAL MIXTURE  $\nabla$   
o.

• MEASURING & MANIPULATING INDIVIDUAL QUANTUM SYSTEMS (SCHRÖDINGER'S KITTENS)

NOBEL PRIZE 2012 :

SERGE HAROCHE & DAVID WINELAND

~> MESOSCOPIC SYSTEMS (SCHRÖDINGER'S KITTENS)

↳ SMALL # TRAPPED IONS OR PHOTONS IN CAVITY

$$|\alpha_0|^2 \sim 10$$

ALLOWS TO STUDY TIME EVOLUTION  
OF SCHRÖDINGER CAT STATE

TOWARDS CLASSICAL STATE

BY DECOHERENCE

SEE e.g. NATURE 455, 510 (2008)

~> MACROSCOPIC SYSTEMS (SCHRÖDINGER'S CAT)

↳ WOULD REQUIRE CONSPIRACY OF  $\sim 10^{27}$  PARTICLES

STATE IS DESTROYED AFTER VERY SHORT TIME

(cf. PENDULUM  $\alpha_0 \sim 10^9$ )