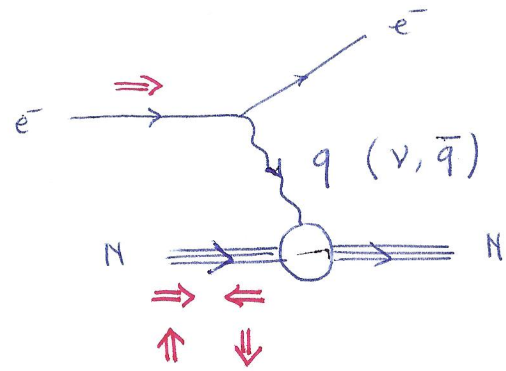
POLARIZED DIS
SPIN OF THE NUCLEON

⇒ **ASYMMETRIES**

- 

$$W^{\mu\nu} = W_S^{\mu\nu} + i\, W_A^{\mu\nu}$$

$$\boxed{W_A^{\mu\nu} = 2\, \varepsilon^{\mu\nu\alpha\beta}\, q_\alpha\, \frac{1}{\nu} \left\{ S_\beta\, g_1 + \left(S_\beta - \frac{S \cdot q}{p \cdot q}\, p_\beta \right) g_2 \right\}}$$

$$W_S^{\mu\nu} = 2 \left\{ \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) F_1 + \frac{1}{p \cdot q} \left(p^\mu - \frac{p \cdot q}{q^2} q^\mu \right) \left(p^\nu - \frac{p \cdot q}{q^2} q^\nu \right) F_2 \right\}$$

POLARIZATION OF e^- (HELICITY)

IS TRANSFERRED TO VIRTUAL PHOTON

↓

3 POLARIZATION STATES

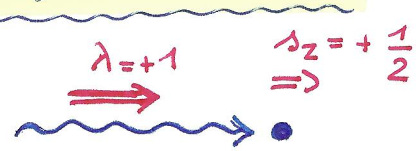
$$\varepsilon^\mu(\lambda = +1) = -\frac{1}{\sqrt{2}} (0, 1, i, 0) \quad \text{RIGHT-HANDED}$$

$$\varepsilon^\mu(\lambda = -1) = \frac{1}{\sqrt{2}} (0, 1, -i, 0) \quad \text{LEFT-HANDED}$$

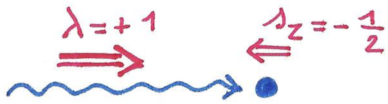
CIRCULAR POL.

$$\varepsilon^\mu(\lambda = 0) = \frac{1}{Q} (|\vec{q}|, 0, 0, \nu) \quad \text{LONGITUDINAL}$$

TRANSVERSE γ^* POLARIZATION



$\sigma_{3/2}$ TOTAL HELICITY : 3/2



$\sigma_{1/2}$ TOTAL HELICITY : 1/2

$$\sigma_{3/2} \sim \varepsilon^*_\mu(\lambda=+1)\,\varepsilon_\nu(\lambda=+1)\,W^{\mu\nu}(s_z=+\tfrac{1}{2})$$

$$\downarrow$$

$$S^\alpha\ (0,0,0,1)$$

PROTON SPIN POL. ALONG z-AXIS IN ITS REST FRAME

$$\sigma_{1/2} \sim \varepsilon^*_\mu(\lambda=+1)\,\varepsilon_\nu(\lambda=+1)\,W^{\mu\nu}(s_z=-\tfrac{1}{2})$$

$$\downarrow$$

$$S^\alpha\ (0,0,0,-1)$$

$\hookrightarrow$ $\varepsilon^*_\mu(\lambda=1)\,\varepsilon_\nu(\lambda=1)\,W_S^{\mu\nu}(s_z=+\tfrac{1}{2})$

$$= -2\,(\varepsilon.\varepsilon^*)\,F_1 = 2F_1$$

$\hookrightarrow$ $\varepsilon^*_\mu(\lambda=1)\,\varepsilon_\nu(\lambda=1)\,W_A^{\mu\nu}(s_z=+\tfrac{1}{2})$

$$= \varepsilon^{*\mu}.\varepsilon^{\nu}.\left[2\,\varepsilon_{\mu\nu\alpha 3}\,q^\alpha\,\frac{1}{\nu}\left\{g_1+g_2\right\} + 2\,\varepsilon_{\mu\nu\alpha 0}\,q^\alpha\,\frac{1}{\nu^2}\,|\vec{q}|\,g_2\right]$$

$$= \varepsilon^{*\mu}\,\varepsilon^{\nu}\left[2\,\varepsilon_{0\mu\nu 3}\,(g_1+g_2) - 2\,\varepsilon_{0\mu\nu 3}\,\frac{|\vec{q}|^2}{\nu^2}\,g_2\right]$$

$$= \left[\frac{1}{2}\,(1)(+i)\,\varepsilon_{0123} + \frac{1}{2}\,(-i)(1)\,\varepsilon_{0213}\right]\cdot\left\{2(g_1+g_2) - 2\,\frac{|\vec{q}|^2}{\nu^2}\,g_2\right\}$$

$$= +i\,2\left\{g_1 - \frac{Q^2}{\nu^2}\,g_2\right\}$$

$$\therefore \quad \varepsilon_\mu(\lambda=+1)\,\varepsilon^*_\nu(\lambda=+1)\,\underbrace{W^{\mu\nu}(s_z=+\tfrac{1}{2})}_{\shortparallel \atop W_S^{\mu\nu}+i\,W_A^{\mu\nu}}$$

$$= 2\left\{ F_1 - \left(g_1 - \frac{Q^2}{\nu^2} g_2\right)\right\}$$

$$\boxed{\begin{aligned} \sigma_{3/2} &\sim 2\left\{ F_1 - \left(g_1 - \frac{Q^2}{\nu^2} g_2\right)\right\} \\ \sigma_{1/2} &\sim 2\left\{ F_1 + \left(g_1 - \frac{Q^2}{\nu^2} g_2\right)\right\} \end{aligned}}$$

$$\Downarrow$$

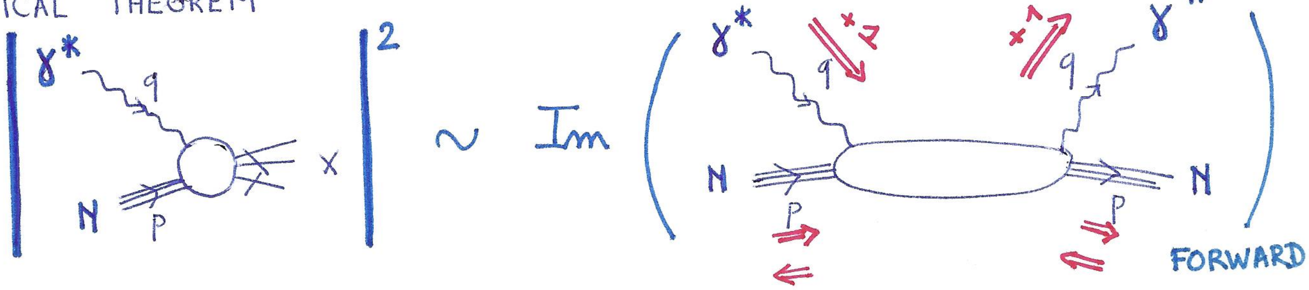
ASYMMETRY

$$\boxed{\frac{\sigma_{1/2} - \sigma_{3/2}}{\sigma_{1/2} + \sigma_{3/2}} = \frac{g_1 - \frac{Q^2}{\nu^2} g_2}{F_1}} \qquad \text{WITH } \frac{Q^2}{\nu^2} = \frac{(2Mx)^2}{Q^2}$$

$$\xrightarrow[Q^2 \gg]{} \frac{g_1}{F_1}$$

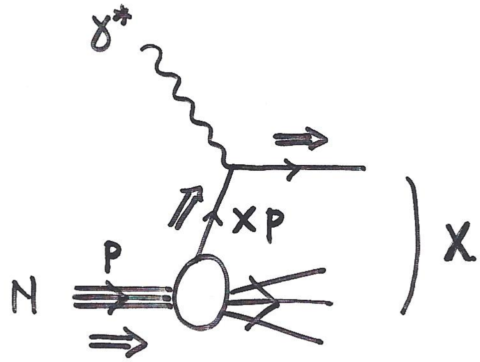
(cf. RESULT IN QUARK PARTON MODEL)

OPTICAL THEOREM

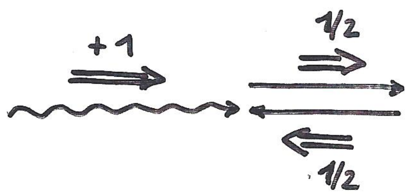


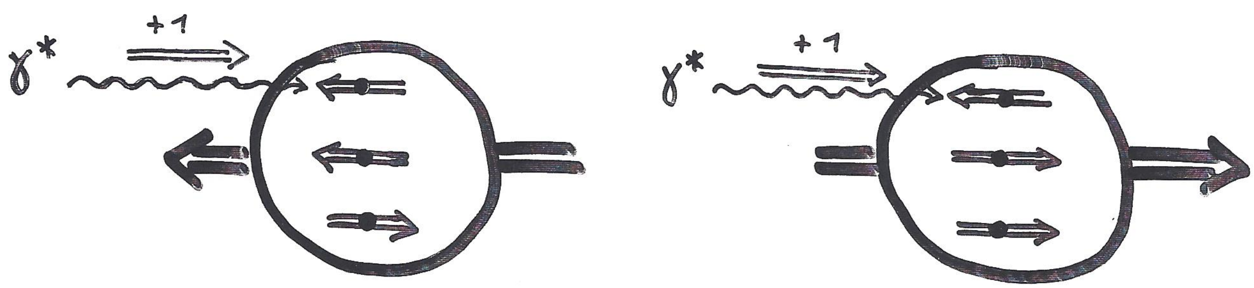
POLARIZED DIS IN QUARK PARTON MODEL

- $\boxed{g_1(x)}$



IN LIMIT $m_q \to 0$: PHOTON & QUARK HELICITIES ARE OPPOSITE





$\sigma_{1/2}$ $\qquad\qquad$ $\sigma_{3/2}$

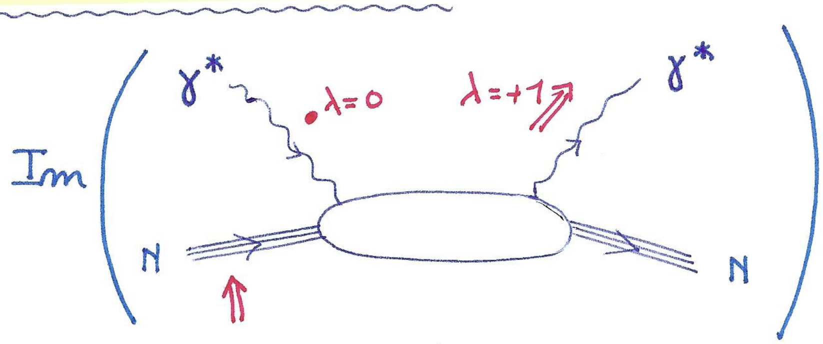
$q_\uparrow(x)$ $\qquad\qquad$ $q_\downarrow(x)$

$$\sigma_{1/2} - \sigma_{3/2} \sim q_\uparrow(x) - q_\downarrow(x) = \Delta q(x)$$

$$\boxed{g_1(x) = \frac{1}{2} \sum_q e_q^2 \, \Delta q(x)}$$

QUARK HELICIT DISTRIBUTION

- **LONGITUDINAL γ^* POLARIZATION**



TRANSVERSE SPIN

$$S^\alpha(0,1,0,0)$$

$$\sigma_{LT} \sim \varepsilon^{\mu *}(\lambda=+1)\,\varepsilon^{\nu}(\lambda=0)\,W_{\mu\nu}$$

$$= i\,\varepsilon^{\mu *}(\lambda=+1)\,\varepsilon^{\nu}(\lambda=0)\,W^{A}_{\mu\nu}$$

$$= 2i\,\varepsilon^{\mu *}(\lambda=+1)\frac{1}{\nu Q}\Big\{|\bar{q}|^2\,\varepsilon_{\mu 0 3\beta}\left[S^\beta g_1 + S^\beta g_2\right]$$

$$+\,\nu^2\,\varepsilon_{\mu 3 0\beta}\left[S^\beta g_1 + S^\beta g_2\right]\Big\}$$

$$\downarrow \quad \beta = 1 \Rightarrow \mu = 2$$

$$= 2i\left(\frac{i}{\sqrt{2}}\right)\frac{1}{\nu Q}\Big\{|\bar{q}|^2\,\underbrace{\varepsilon_{2031}}_{-1} + \nu^2\,\underbrace{\varepsilon_{2301}}_{+1}\Big\}(g_1+g_2)$$

$$= \sqrt{2}\,\frac{Q}{\nu}\,(g_1+g_2)$$

$$\sigma_{LT} \sim \sqrt{2}\,\frac{2Mx}{Q}\,(g_1+g_2)$$

ASYMMETRY

$$\boxed{\frac{2\sigma_{LT}}{\sigma_{1/2}+\sigma_{3/2}} = \sqrt{2}\left(\frac{2Mx}{Q}\right)\cdot\frac{g_1+g_2}{2F_1}}$$

$\Rightarrow$ NUCLEON SPIN STRUCTURE IN NON-RELATIVISTIC CONSTITUENT QUARK MODEL

- SU(6) SPIN-FLAVOR WAVEFUNCTION

$$|P_\uparrow\rangle = \frac{1}{\sqrt{6}}\left\{2\,|u_\uparrow u_\uparrow d_\downarrow\rangle - |u_\uparrow u_\downarrow d_\uparrow\rangle - |u_\downarrow u_\uparrow d_\uparrow\rangle\right\}$$

$+$ SU(6) PERMUTATIONS

- $\langle u_\uparrow\rangle = \langle P_\uparrow|\,\frac{1}{2}(1+\hat{\sigma}_3)\,\frac{1}{2}(1+\hat{\tau}_3)\,|P_\uparrow\rangle$

$$\boxed{\begin{array}{l}\hat{\sigma}_3 \equiv \sum_{i=1}^{3}\hat{\sigma}_3^{(i)} \\ \quad\hookrightarrow \text{FOR EACH OF 3 QUARKS} \\ \hat{\tau}_3 \equiv \sum_{i=1}^{3}\tau_3^{(i)}\end{array}}$$

$$\Delta u = \langle u_\uparrow\rangle - \langle u_\downarrow\rangle$$

$$= \frac{1}{2}\langle P_\uparrow|\,\hat{\sigma}_3\,(1+\hat{\tau}_3)\,|P_\uparrow\rangle$$

$$= \frac{1}{2}\left\{\underbrace{\langle\hat{\sigma}_3\rangle}_{\frac{1}{6}(4+4-2)} + \underbrace{\langle\hat{\sigma}_3\hat{\tau}_3\rangle}_{\frac{1}{6}(4+4+2)}\right\}$$

$$\frac{1}{6}(\underset{\text{QUARK 1}}{4} + \underset{\text{QUARK 2}}{4} - \underset{\text{QUARK 3}}{2})$$

$$\Delta u = \frac{1}{2}\left\{1+\frac{5}{3}\right\} \Rightarrow \boxed{\Delta u = \frac{4}{3}}$$

TOTAL NUCLEON SPIN CONTRIBUTION CARRIED BY U-QUARK

- $\langle\Delta d\rangle = \frac{1}{2}\left\{\langle\hat{\sigma}_3\rangle - \langle\hat{\sigma}_3\hat{\tau}_3\rangle\right\}$

$$= \frac{1}{2}\left\{1-\frac{5}{3}\right\} \Rightarrow \boxed{\Delta d = -\frac{1}{3}}$$

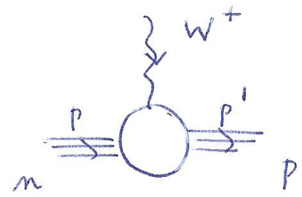
- IN NON-REL. QUARK MODEL $\underline{\underline{\Delta\Sigma \equiv \Delta u + \Delta d = 1}}$

AXIAL CURRENT

- $$\langle p | \tfrac{1}{2} (\bar{u} \gamma^\mu \gamma_5 u - \bar{d} \gamma^\mu \gamma_5 d) | p \rangle$$

$$\equiv \bar{N} \; G_A(t) \, \gamma^\mu \gamma_5 \, N \; + \text{ TERM WHICH VANISHES FOR } p' = p$$

AXIAL FORM FACTOR



$t \equiv (p' - p)^2$

$G_A(t=0) \equiv g_A$ AXIAL COUPLING CONSTANT

EXPERIMENTALLY : $g_A \approx 1.267$ FROM n - BETA DECAY

$$\hookrightarrow \langle p_\uparrow | \tfrac{1}{2} (\bar{u} \gamma^i \gamma_5 u - \bar{d} \gamma^i \gamma_5 d) | p_\uparrow \rangle_{t=0}$$

$$= g_A \; \bar{N}(p, \uparrow) \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} \begin{pmatrix} 0 & 1\!\!1 \\ 1\!\!1 & 0 \end{pmatrix} N(p, \uparrow)$$

$$= g_A \; N^+(p, \uparrow) \begin{pmatrix} \sigma^i & 0 \\ 0 & \sigma^i \end{pmatrix} N(p, \uparrow)$$

cfr. SPIN VECTOR OPERATOR

$\Downarrow$

$$g_A = \langle \hat{\sigma}_3 \, \hat{\tau}_3 \rangle$$

$$\boxed{g_A = \frac{5}{3} = \Delta u - \Delta d} > 1.267 \; !$$

- RELATIVISTIC QUARK MODEL

$\Delta u \approx 1.0$, $\Delta d \approx -0.25$ $\Rightarrow$ $\Delta \Sigma \approx 0.75$

FIRST MOMENT OF g_1

- $$g_1^p(x) = \frac{1}{2}\left\{\frac{4}{9}(\Delta u + \Delta\bar{u}) + \frac{1}{9}(\Delta d + \Delta\bar{d}) + \frac{1}{9}(\Delta s + \Delta\bar{s}\right.$$

$$g_1^n(x) = \frac{1}{2}\left\{\frac{4}{9}(\Delta d + \Delta\bar{d}) + \frac{1}{9}(\Delta u + \Delta\bar{u}) + \frac{1}{9}(\Delta s + \Delta\bar{s})\right.$$

$$\boxed{\Gamma_1^{p,n} \equiv \int_0^1 dx \; g_1^{p,n}(x)}$$

- <u>$SU(3)_f$ AXIAL CHARGES</u>

$$\Delta q \equiv \int_0^1 dx \; \Delta q(x)$$

$$a_0 = (\Delta u + \Delta\bar{u}) + (\Delta d + \Delta\bar{d}) + (\Delta s + \Delta\bar{s}) \equiv \Delta\Sigma$$

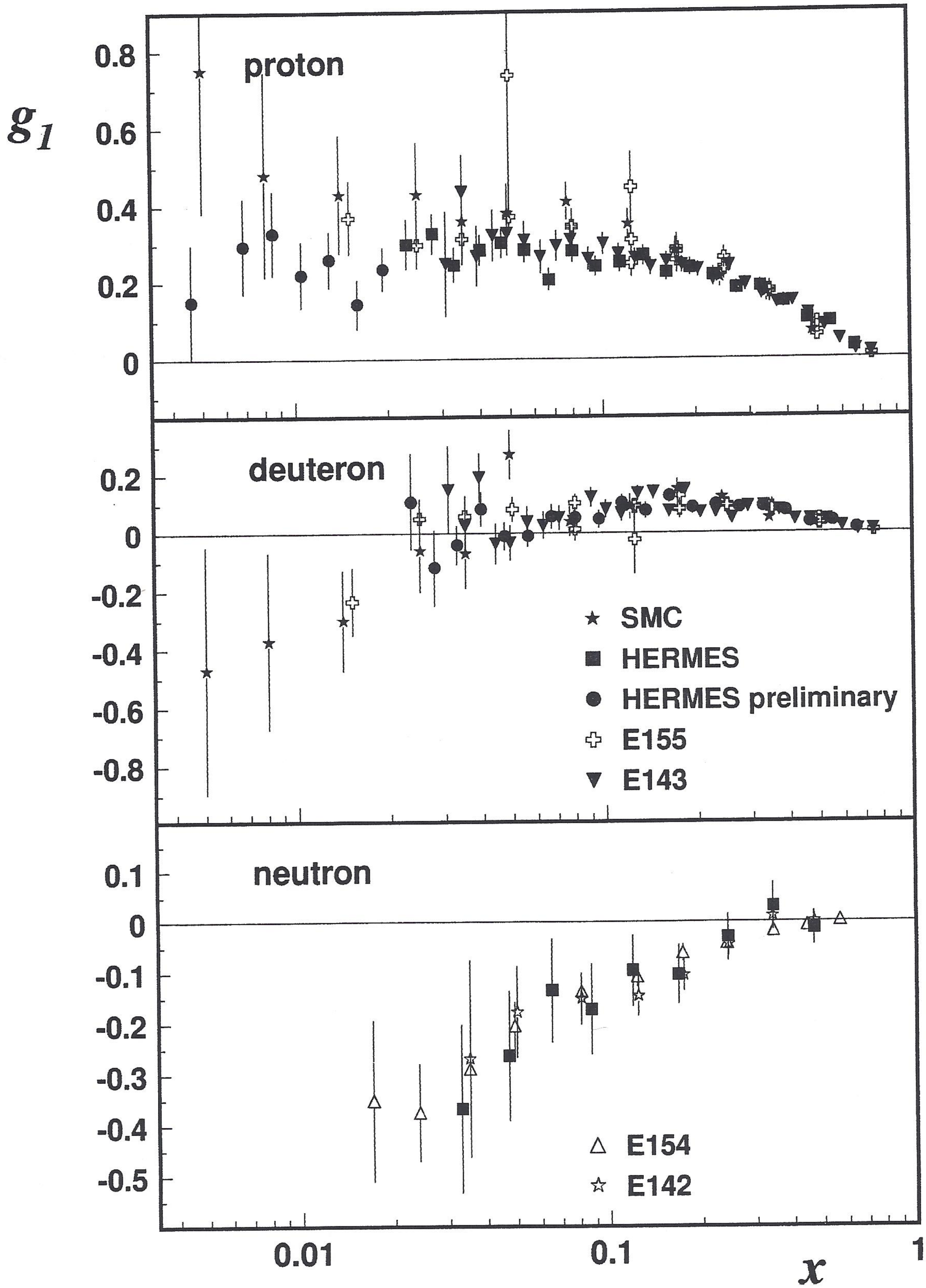
$$a_3 = (\Delta u + \Delta\bar{u}) - (\Delta d + \Delta\bar{d})$$

$$a_8 = (\Delta u + \Delta\bar{u}) + (\Delta d + \Delta\bar{d}) - 2(\Delta s + \Delta\bar{s})$$

n β-DECAY $\Rightarrow$ $a_3 = g_A \simeq 1.267$

HYPERON β-DECAY $\Rightarrow$ $a_8 \simeq 0.58 \pm 0.03$

g_1 STRUCTURE FUNCTIONS of p, d, and n



BJORKEN SUM RULE

- $$\boxed{\begin{aligned}\Gamma_1^p - \Gamma_1^n &= \frac{1}{6}\int_0^1 dx \left\{(\Delta u + \Delta\bar{u}) - (\Delta d + \Delta\bar{d})\right\} \\ &= \frac{1}{6} a_3 \\ &= \underbrace{\frac{1}{6} g_A}_{\simeq 0.21} \left[1 - \frac{\alpha_s(Q^2)}{\pi} + O(\alpha_s^2)\right]\end{aligned}}$$

BASED ON ISOSPIN SYMMETRY & CURRENT ALGEBRA

- **EXPERIMENT**

(E155 '00) $\Gamma_1^p - \Gamma_1^n (Q^2 = 5\ GeV^2) = 0.176 \pm 0.003 \pm 0.007$

THEORY : $\Gamma_1^p - \Gamma_1^n (Q^2 = 5\ GeV^2) = 0.181 \pm 0.005$ ✓

ELLIS - JAFFE SUM RULES

- $$\boxed{\Gamma_1^{p,n} = \frac{1}{12}\left\{\frac{4}{3}a_0 \pm a_3 + \frac{1}{3}a_8\right\}}$$

- ELLIS - JAFFE : ASSUME $\Delta s = \Delta\bar{s} = 0$

$$\Downarrow$$

$$a_0 = a_8 \iff \Delta\Sigma \simeq 0.6$$

$$\Gamma_{1,EJ}^{p,n} = \frac{1}{12}\left\{\frac{5}{3}a_8 \pm a_3\right\}$$

$$Q^2 = 5\,GeV^2 \nearrow \Gamma_{1,EJ}^{p} = 0.163 \pm 0.004$$

$$\searrow \Gamma_{1,EJ}^{n} = -0.019 \pm 0.004$$

- EXPERIMENT (SLAC, SMC, HERMES)

(E155)

$$\Gamma_1^p(Q^2 = 5\,GeV^2) = 0.118 \pm 0.004 \pm 0.007$$

$$\Gamma_1^n(Q^2 = 5\,GeV^2) = -0.058 \pm 0.005 \pm 0.008$$

ELLIS - JAFFE SUM RULES **NOT** VERIFIED

EXPERIMENT $\Rightarrow$ $\boxed{\Delta\Sigma \simeq 0.3}$

NOTE HOWEVER SCHEME DEPENDE[NT] AT NLO

Where does main part of nucleon spin come from ?

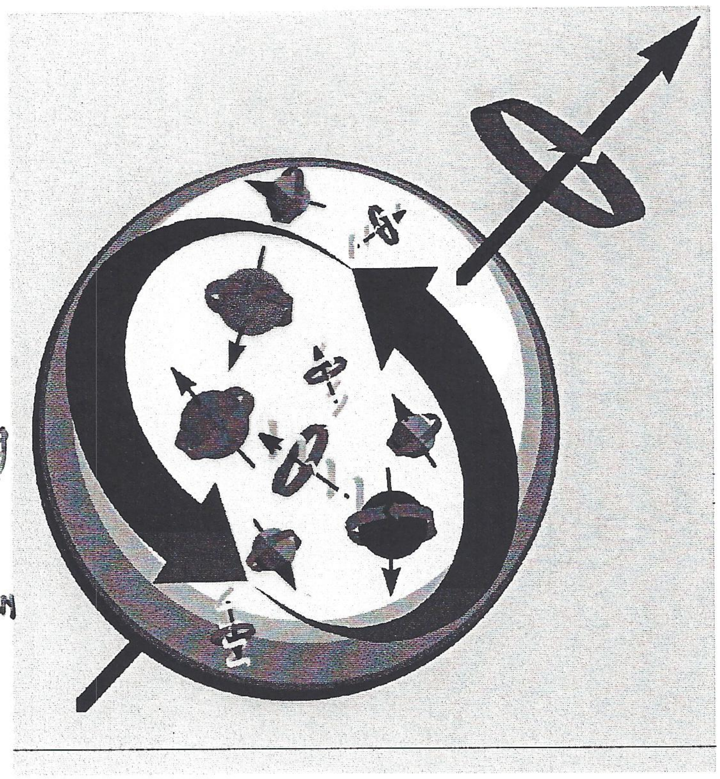
QUARK GLUON

$$\frac{1}{2} = J_q + J_g$$

$$= \Delta\Sigma + L_q + \Delta G + L_g$$

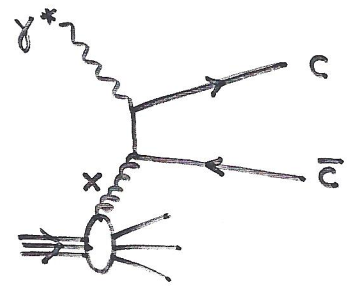
QUARK SPIN, GLUON SPIN

QUARK, GLUON ORBITAL



$\Longrightarrow$ GLUON contribution

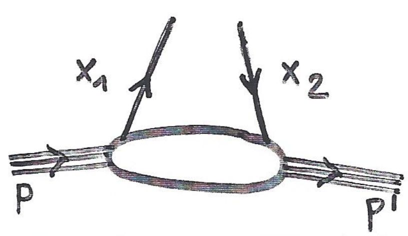
ΔG



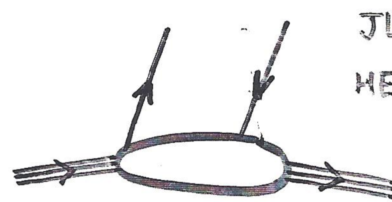
HERMES
COMPASS
SLAC
RHIC SPIN

$\Longrightarrow$ ORBITAL contribution
accessible through Generalized Parton Distributions (GPD)

L_q



OR



JLAB
HERMES

GPDs : 'microsurgery' of nucleon at quark level

GLUON HELICITY DISTRIBUTION

$$\boxed{J_g = \Delta G + L_g}$$

(LIGHT-CONE GAUGE)
↓
PARTONIC INTERPRETATION

⇒ ΔG FROM QCD SCALE EVOLUTION

NLO :

$$\boxed{\begin{aligned} g_1(x,Q^2) &= \frac{1}{2} \sum_q e_q^2 \, C_q(x,\alpha_s) \otimes \Delta q(x,Q^2) \\ &\quad + \frac{1}{N_f} \, C_g(x,\alpha_s) \otimes \Delta G(x,Q^2) \end{aligned}}$$

e.g. 

C_q, C_g : WILSON COEFFICIENTS, CALCULATED IN PERTURBATIVE QCD

FACTORIZATION SCHEME DEPENDENCE ($\overline{MS}$, AB, ...)

$$C_q(x,\alpha_s) = \delta(1-x) + \frac{\alpha_s(Q^2)}{2\pi} C_q^{(1)} + \dots$$

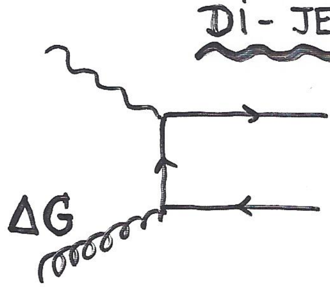
$$C_g(x,\alpha_s) = 0 + \frac{\alpha_s(Q^2)}{2\pi} C_g^{(1)} + \dots$$

DISADVANTAGE : MEASURED RANGE IN Q^2 IS LIMITED
↪ IMPRECISE EXTRACTION

⇒ DIRECT MEASUREMENTS OF ΔG (HERMES, COMPASS, RHIC)

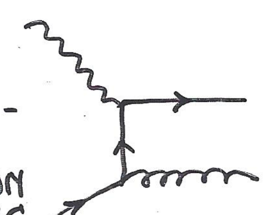
DI-JET PRODUCTION IN DIS

⇙ ⇘

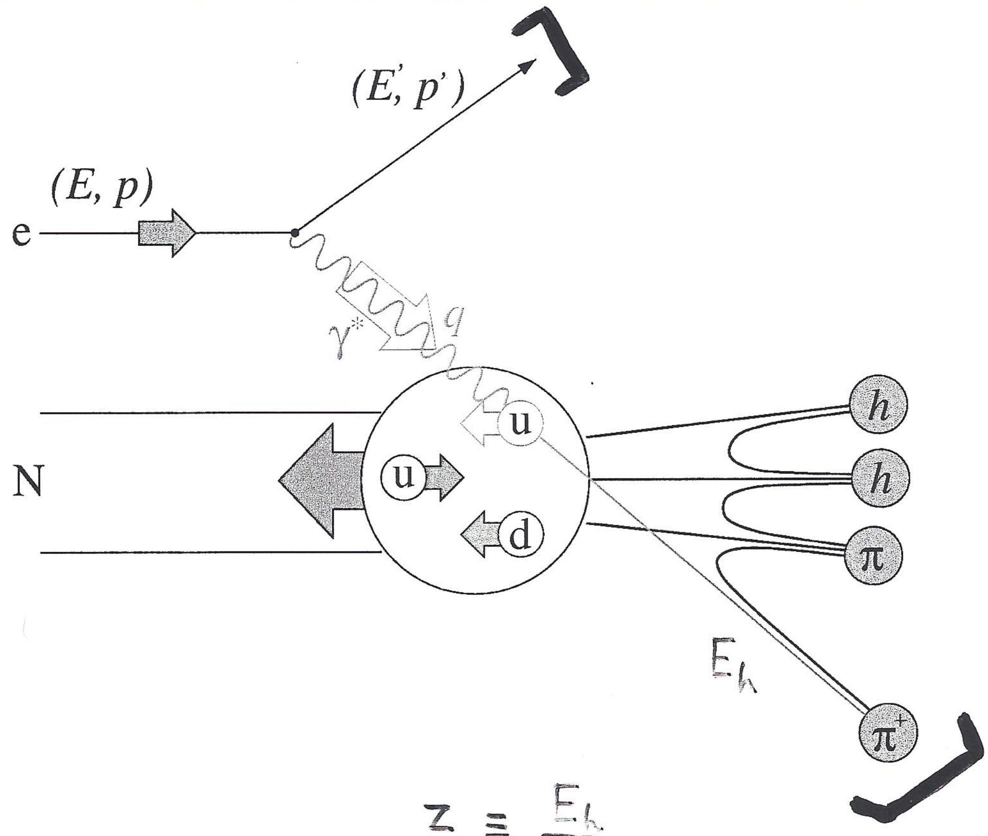


PHOTON-GLUON FUSION

PHOTON-QUARK COMPTON SCATTERING



POLARIZED SEMI-INCLUSIVE DIS



$z \equiv \frac{E_h}{\nu}$

DETECTED

$(z > 0.2)$

VIRTUAL PHOTON NUCLEON ASYMMETRY

$$A_1^h(x, Q^2) = (\sigma_{1/2} - \sigma_{3/2}) \,/\, (\sigma_{1/2} + \sigma_{3/2})$$

$$\approx \frac{\sum_q e_q^2 \; \Delta q(x, Q^2) \cdot \int_{z_{min}}^1 dz\, D_q^h(z, Q^2)}{\sum_q e_q^2 \; q(x, Q^2) \cdot \int_{z_{min}}^1 dz\, D_q^h(z, Q^2)}$$

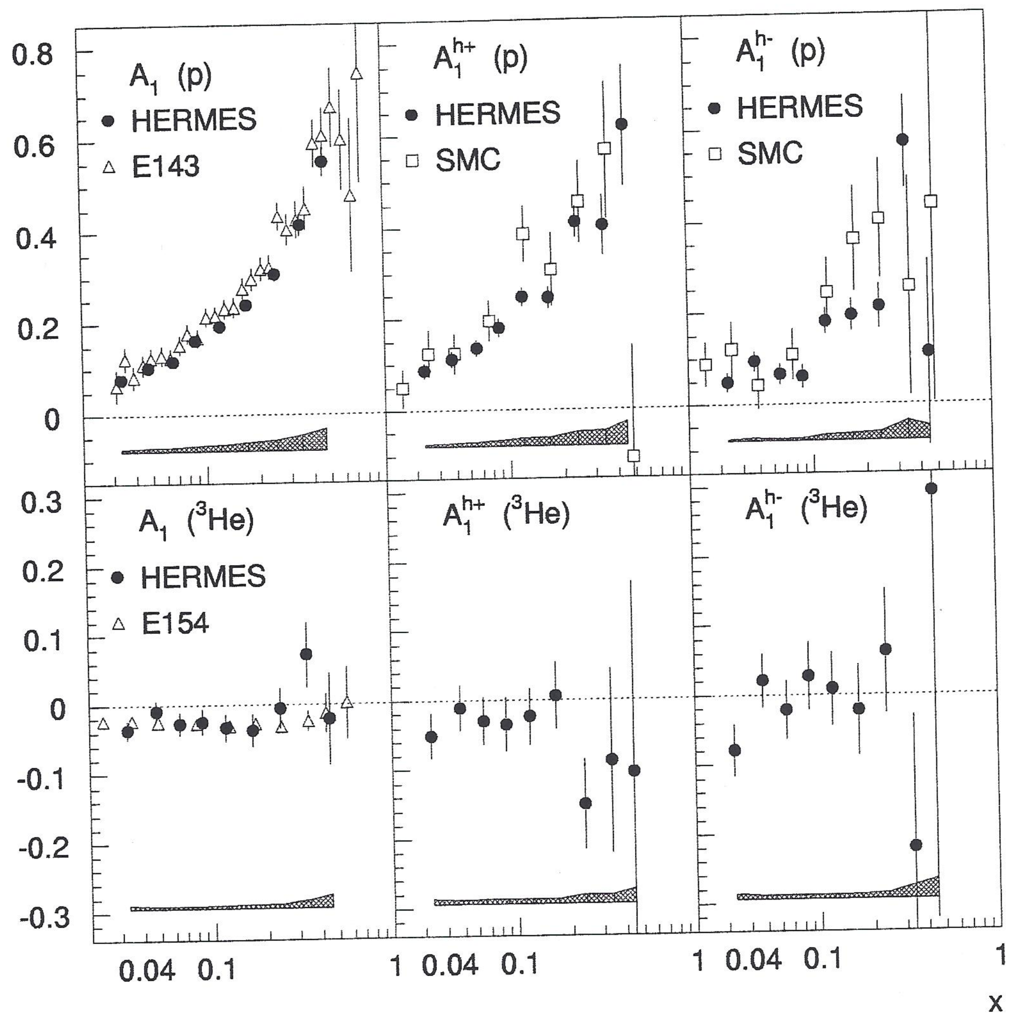
FRAGMENTATION FUNCTION (LUND MODEL)

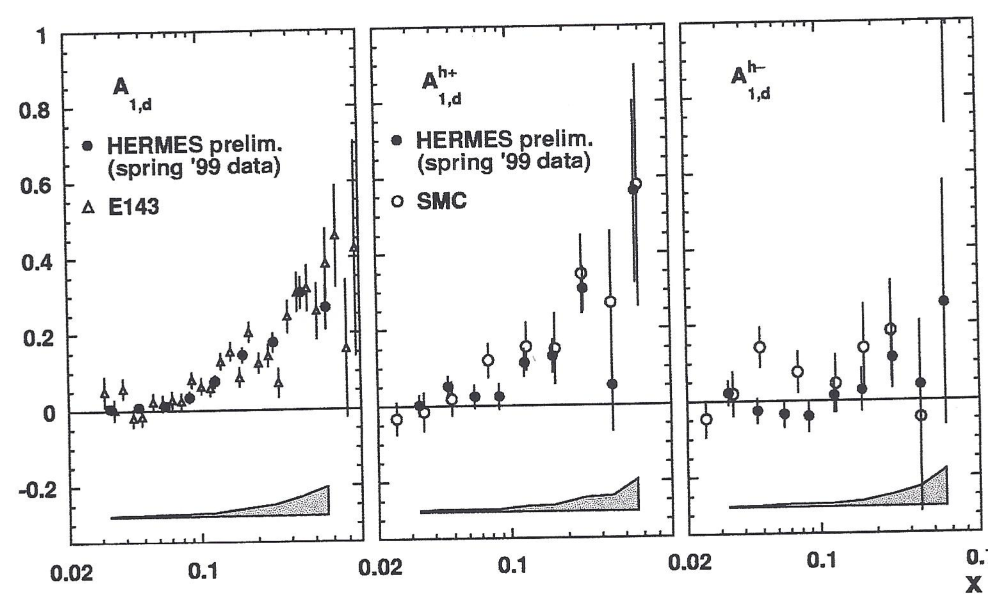
$$= \sum_q P_q^h(x) \frac{\Delta q(x)}{q(x)}$$

'PURITY' FUNCTION: PROBABILITY THAT HADRON ORIGINATES FROM THE SCATTERING OFF A QUARK q!

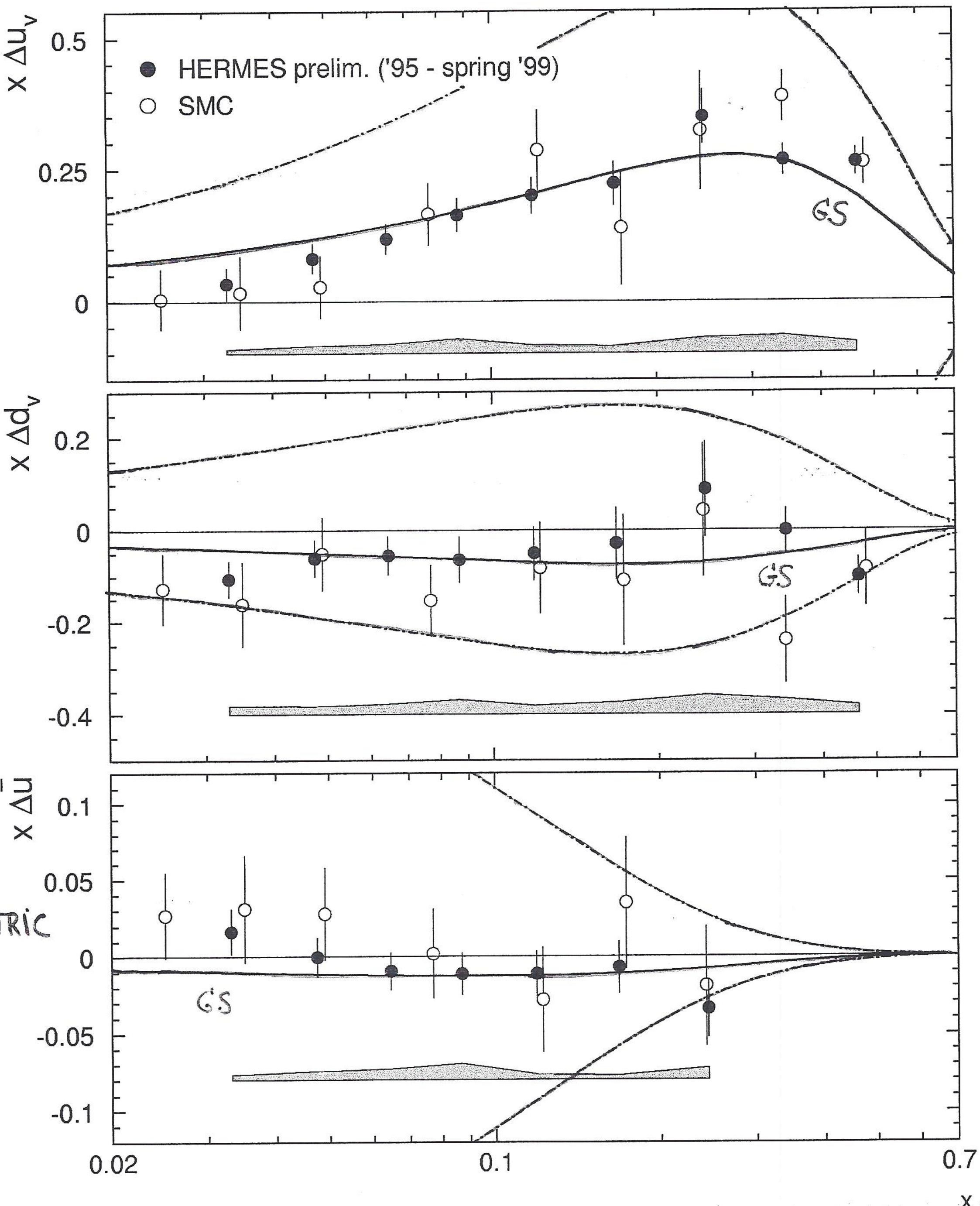
(GENERATED BY MC)

SEMI-INCLUSIVE ASYMMETRIES





POLARIZED VALENCE and SEA Quark Distributions in the PROTON



SYMMETRIC SEA

$\Delta\bar{u} = \Delta\bar{d}$

$= \Delta\bar{s}$

SMC DATA : EVOLVED TO $Q^2 = 2.5$ GeV^2

—— GS : GEHRMANN - STIRLING PARAMETRIZATION (LO)

—— POSITIVITY LIMITS $\Delta q(x) = q(x)$

SEPARATED QUARK SPIN DISTRIBUTIONS

- **SEMI-INCLUSIVE DIS** (SMC, HERMES)

HERMES RESULTS AT $Q^2 = 2.5 \text{ GeV}^2$

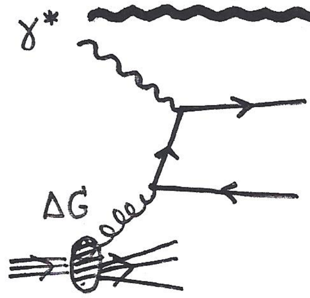
$$\boxed{\begin{aligned} \Delta u_v &= 0.57 \pm 0.05 \pm 0.08 \\ \Delta d_v &= -0.22 \pm 0.11 \pm 0.13 \\ \Delta \bar{u} &= -0.01 \pm 0.02 \pm 0.03 \end{aligned}}$$

$\Downarrow$

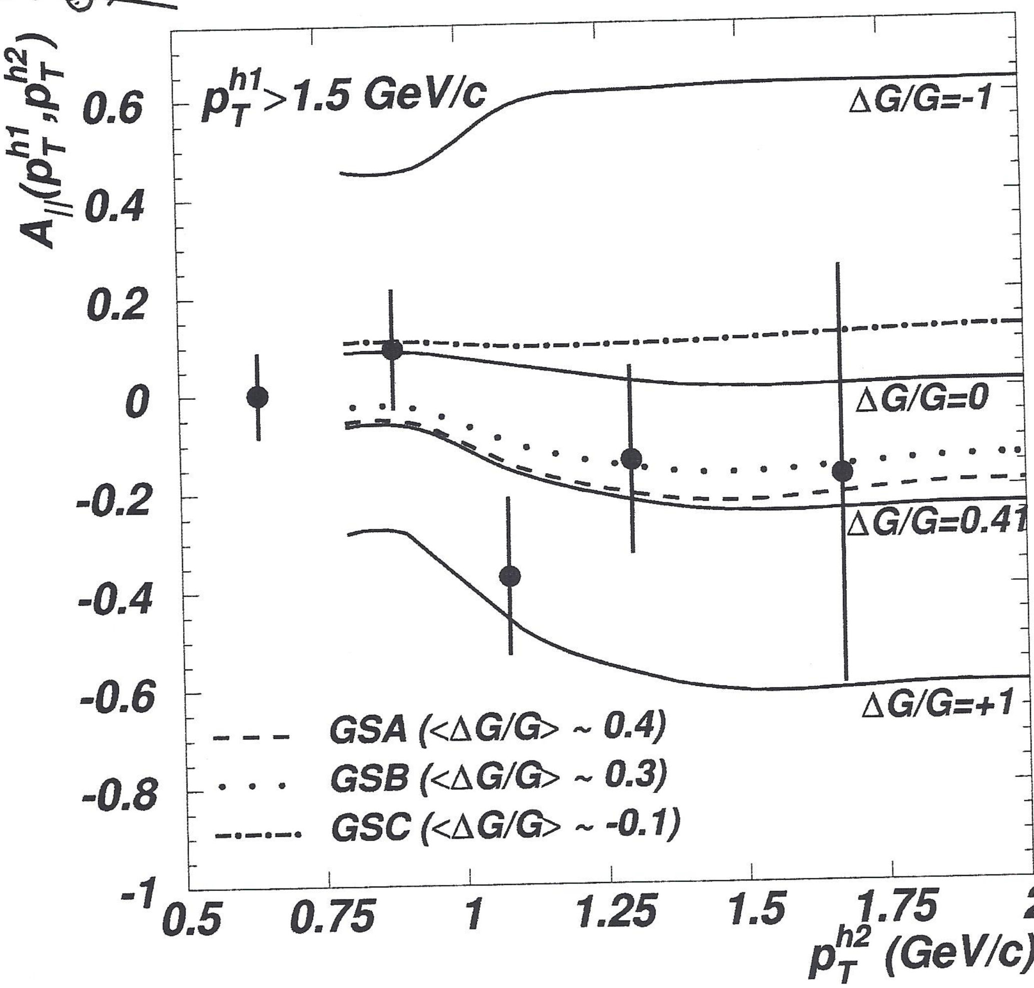
$$\Delta\Sigma \simeq 0.3$$

POLARIZED GLUON Distribution in the PROTON

"DI-JET" PRODUCTION IN DIS



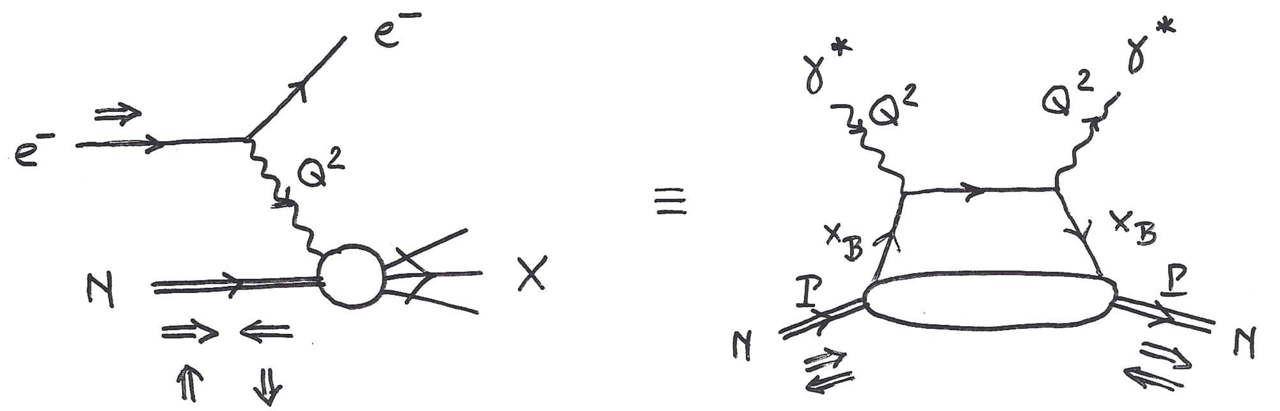
AT LEAST
2 HADRONS
- OF OPPOSITE CHARGE
- AT HIGH P_T $\quad (M_{\pi\pi} > 1\,\mathrm{GeV})$

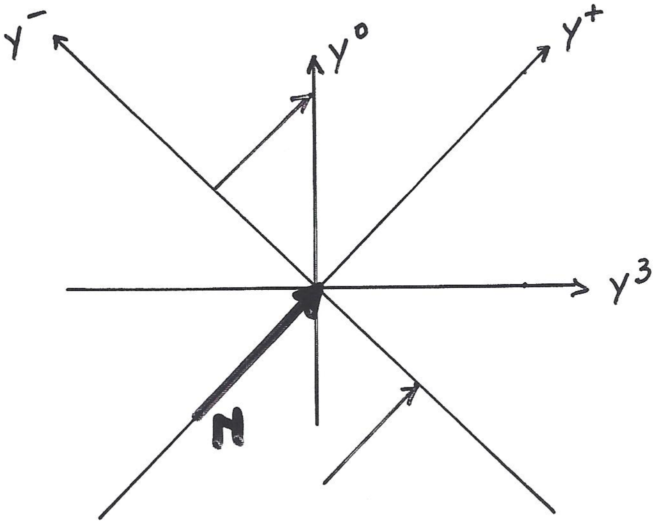


(HERMES PRL '00) $\boxed{\left\langle \frac{\Delta G}{G} \right\rangle = 0.41 \pm 0.18\,(\text{stat.}) \pm 0.03\,(\text{syst.})}$

WITH $\langle x_G \rangle = 0.17$

WANDZURA - WILCZEK RELATION FOR g_2



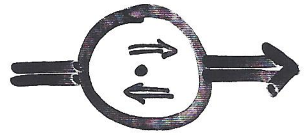


$$S^\mu = (S.n)P^\mu + S_\perp^\mu$$

$$(S.n)\, g_1(x) = P^+ \int \frac{dy^-}{2\pi}\, e^{ixP^+y^-} \langle PS|\, \bar{\psi}\!\left(\tfrac{y}{2}\right) \not{n}\, \gamma_5\, \psi\!\left(-\tfrac{y}{2}\right) |PS\rangle \Big|_{\substack{y^+=0\\ y_\perp=0}}$$

$$S_\perp^\mu\, g_T(x) = P^+ \int \frac{dy^-}{2\pi}\, e^{ixP^+y^-} \langle PS|\, \bar{\psi}\!\left(\tfrac{y}{2}\right) \gamma_\perp^\mu \gamma_5\, \psi\!\left(-\tfrac{y}{2}\right) |PS\rangle \Big|_{\substack{y^+=0\\ y_\perp=0}}$$

$$g_T \equiv g_1 + g_2$$

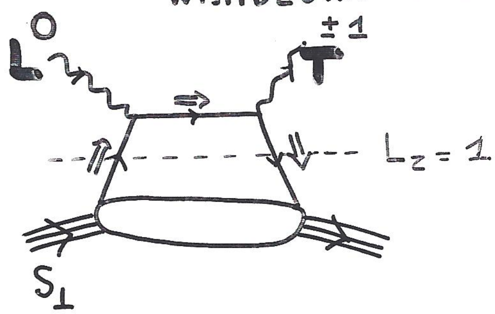
- g_1 : TWIST - 2  $\sim \Delta q(x)$

- g_2 : TWIST - 2 + TWIST - 3

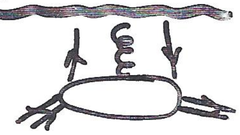
$$\boxed{g_2(x, Q^2) = g_2^{WW}(x, Q^2) + \bar{g}_2(x, Q^2)}$$

TWIST - 2

WANDZURA - WILCZEK



- QUARK MASS TE
- TWIST-3 QUARK-GLUO CORRELATIONS IN NUCLEON



$$\Rightarrow \quad \boxed{g_2^{WW}(x_B, Q^2) = -g_1(x_B, Q^2) + \int_{x_B}^{1} dx \, \frac{1}{x} \, g_1(x, Q^2)}$$

(WANDZURA , WILCZEK 1977)

$\Rightarrow$ DEVIATION $\bar{g}_2$: MEASURE FOR SIZE OF TWIST - 3 QUARK - GLUON MATRIX E

MOMENT $\boxed{d_2 \equiv 3 \int_0^1 dx \; x^2 \; \bar{g}_2(x)}$ $\qquad d_2^{WW} = 0$

$$= \int_0^1 dx \; x^2 \left\{ 3\, g_2(x) + 2\, g_1(x) \right\}$$

Experimental results for g_2