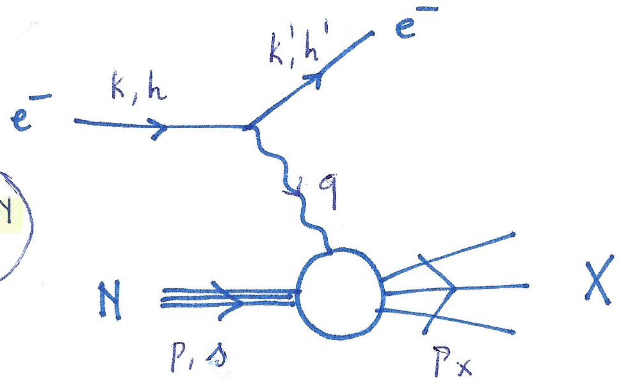
DEEP INELASTIC LEPTON-NUCLEON SCATTERING.

PARTON MODEL

⇒ ELECTRON-NUCLEON SCATTERING : KINEMATICS

ONE-PHOTON EXCHANGE



$h\,(h')$: INITIAL (FINAL) e^- HELICITIES

$h = \pm\frac{1}{2}$, $h' = \pm\frac{1}{2}$

s : INITIAL NUCLEON SPIN

$s = \pm\frac{1}{2}$

$$k\,(E_e, \bar{k}) \qquad E_e = |\bar{k}|$$
$$k'\,(E_e', \bar{k}') \qquad E_e' = |\bar{k}'|$$
$\Big\}$ $m_e \approx 0$

LAB SYSTEM $\quad p\,(M, 0)$

$$q \equiv k - k' = (\nu, \bar{q})$$

$$q^2 = \nu^2 - \bar{q}^2$$
$$= -2\,k.k' = -4E_eE_e' \sin^2\frac{\theta}{2} < 0 \qquad \text{SPACELIKE } \gamma$$

$$-q^2 \equiv +Q^2 > 0$$

⇒ ELECTRON - NUCLEON SCATTERING : CROSS SECTION

IN LAB SYSTEM

→ SUM OVER FINAL LEPTON HELICITY

$$d\sigma = \frac{1}{v_{rel}} \cdot \frac{1}{(2E_e)(2M)} \sum_{h'} \frac{d^3\bar{k}'}{(2\pi)^3\, 2E'_e} \sum_X (2\pi)^4 \delta^4(q+p-p_X)$$

$$\cdot \left| \bar{U}(k'h')\gamma_\nu U(kh) \cdot \frac{e^2}{Q^2} \cdot \langle X | J^\nu(0) | p s\rangle \right|^2$$

$$\downarrow \quad v_{rel} = \frac{|\bar{k}|}{E_e} + \frac{|\bar{P}|}{E_N} = \frac{|\bar{k}|}{E_e} \approx 1 \quad \text{(INITIAL FLUX)}$$

$$d\sigma = \frac{(4\pi\alpha)^2}{Q^4} \cdot \frac{1}{4ME_e} \frac{d\Omega'_e \cdot dE'_e\, E'_e}{(2\pi)^3 \; 2} \cdot$$

$$\alpha \equiv \frac{e^2}{4\pi} \simeq \frac{1}{137}$$

$$\cdot \sum_{h'} \bar{U}(kh)\gamma_\mu U(k'h')\,\bar{U}(k'h')\gamma_\nu U(kh)$$

$$\cdot \sum_X (2\pi)^4 \delta^4(q+p-p_X) \cdot \langle ps | J^{\mu +}(0) | X\rangle\langle X | J^\nu(0) | ps\rangle$$

↓

$$\left(\frac{d\sigma}{d\Omega'_e\, dE'_e}\right)^{LAB} = \frac{\alpha^2}{Q^4} \frac{1}{2M} \cdot \frac{E'_e}{E_e} \cdot L_{\mu\nu} \cdot W^{\mu\nu}$$

LEPTON TENSOR $\quad L_{\mu\nu} \equiv \sum_{h'} \bar{U}(kh)\gamma_\mu U(k'h')\,\bar{U}(k'h')\gamma_\nu U(kh)$

HADRON TENSOR $\quad W^{\mu\nu} \equiv \frac{1}{2\pi} \sum_X (2\pi)^4 \delta^4(q+p-p_X) \cdot \langle ps | J^{\mu +}(0) | X\rangle\langle X | J^\nu(0) | ps\rangle$

- LEPTON TENSOR

$$L_{\mu\nu} \equiv \sum_{h'} \bar{u}(k\,h)\,\gamma_\mu\, u(k'h')\,\bar{u}(k'h')\,\gamma_\nu\, u(k\,h)$$

$\downarrow$ $\quad \sum_{h'} u(k'h')\,\bar{u}(k'h') = \not{k}'$

$$= \mathrm{Tr}\left\{ \gamma_\mu \not{k}' \gamma_\nu\, u(k\,h)\,\bar{u}(k\,h) \right\}$$

$$u(k\,h)\,\bar{u}(k\,h) = \left(\frac{1+(2h)\gamma_5}{2}\right) \sum_{h''} u(k\,h'')\,\bar{u}(k\,h'') = \left(\frac{1+2h\,\gamma_5}{2}\right)\not{k}$$

$\Uparrow$ HELICITY PROJECTOR

$$L_{\mu\nu} = \mathrm{Tr}\left\{ \gamma_\mu \not{k}' \gamma_\nu \left(\frac{1+(2h)\gamma_5}{2}\right)\not{k} \right\}$$

$$= \frac{1}{2}\underbrace{\mathrm{Tr}\left\{\gamma_\mu \not{k}' \gamma_\nu \not{k}\right\}}_{4\left\{k'_\mu k_\nu + k'_\nu k_\mu - k.k'g_{\mu\nu}\right\}} - \frac{(2h)}{2}\underbrace{\mathrm{Tr}\left\{\gamma_5\, \gamma_\mu \not{k}' \gamma_\nu \not{k}\right\}}_{4i\,\varepsilon_{\mu\beta\nu\alpha}\, k'^\beta k^\alpha \;=\; -4i\,\varepsilon_{\mu\nu\alpha\beta}\, k^\alpha q^\beta}$$

$(\varepsilon_{0123} = +1)$

$$L_{\mu\nu} = L^S_{\mu\nu} + i\,L^A_{\mu\nu}$$

SYMMETRIC PART: $\quad L^S_{\mu\nu} = 2\left\{k_\mu k'_\nu + k_\nu k'_\mu - k.k'\,g_{\mu\nu}\right\}$

ANTI-SYMMETRIC PART: $\quad L^A_{\mu\nu} = 2(2h)\,\varepsilon_{\mu\nu\alpha\beta}\, k^\alpha q^\beta$

HADRON TENSOR

$$W^{\mu\nu} \equiv \frac{1}{2\pi} \sum_X (2\pi)^4 \delta^4(q+p-p_X) \langle p s | J^{\mu\dagger}(0) | X \rangle \langle X | J^\nu(0) | p s \rangle$$

$W^{\mu\nu}$ CAN BE PARAMETRIZED IN TERMS OF 4 STRUCTURE FUNCTIONS

$$\boxed{W^{\mu\nu} = W_S^{\mu\nu} + i\, W_A^{\mu\nu}}$$

$W_S^{\mu\nu}$: SYMMETRIC i.e. SPIN INDEPENDENT

$W_A^{\mu\nu}$: ANTI-SYMMETRIC (i.e. SPIN DEPENDENT)

$\Rightarrow$ $W_S^{\mu\nu}$ 2 INDEPENDENT 4 MOMENTA q^μ, p^μ

TENSORS $q^\mu q^\nu$, $p^\mu p^\nu$, $q^\mu p^\nu$, $q^\nu p^\mu$, $g^{\mu\nu}$

+ GAUGE INVARIANCE $q_\mu W^{\mu\nu} = q_\nu W^{\mu\nu} = 0$

$\Downarrow$

ONLY 2 TENSORS SURVIVE

$$\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) \quad \text{AND} \quad \left(p^\mu - \frac{p.q}{q^2} q^\mu \right)\left(p^\nu - \frac{p.q}{q^2} q^\nu \right)$$

$$\boxed{\frac{1}{2M} W_S^{\mu\nu} \equiv \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) W_1(Q^2,\nu) + \frac{1}{M^2}\left(p^\mu - \frac{p.q}{q^2} q^\mu \right)\left(p^\nu - \frac{p.q}{q^2} q^\nu \right) W_2(Q^2,\nu)}$$

W_1 & W_2 → DEPEND ON Q^2, ν

→ UNPOLARIZED NUCLEON STRUCTURE FUNCTIONS

⇒ $W_A^{\mu\nu}$ DEPENDS LINEARLY ON NUCLEON SPIN

S^μ : 4-VECTOR : NUCLEON POLARIZATION VECTOR

IN NUCLEON REST FRAME $S^\mu = (0, \bar{n})$

$$\bar{n}^2 = 1$$

$\bar{n}$: AXIS ALONG WHICH NUCLEON SPIN IS QUANTIZED

$$\begin{cases} S_\mu S^\mu = -\bar{n}^2 = -1 \\ S_\mu \cdot p^\mu = 0 . M - \bar{n} . \bar{0} = 0 \end{cases}$$

→ LORENTZ INV. (HOLD IN ANY FRAME)

$$S_\mu S^\mu = -1 \quad , \quad S_\mu p^\mu = 0$$

$W_A^{\mu\nu}$ → DEPENDS LINEARLY ON S

→ SATISFIES $q_\mu W_A^{\mu\nu} = q_\nu W_A^{\mu\nu} = 0$

$W_A^{\mu\nu}$ INVOLVES → $\varepsilon^{\mu\nu\alpha\beta} q_\alpha S_\beta$

→ $\varepsilon^{\mu\nu\alpha\beta} q_\alpha P_\beta . (S . q)$

OR INSTEAD OF

$$\varepsilon^{\mu\nu\alpha\beta}\, q_\alpha\, p_\beta\, (S.q)$$

WE CAN USE

$$\varepsilon^{\mu\nu\alpha\beta}\, q_\alpha \underbrace{\left(S_\beta - \frac{S.q}{p.q}\, p_\beta \right)}$$

'TRANSVERSE' SPIN VECTOR

$$\equiv (S_\perp)_\beta \qquad (q_\beta \cdot S_\perp^\beta = 0)$$

$$\frac{1}{2M} W_A^{\mu\nu} \equiv \varepsilon^{\mu\nu\alpha\beta}\, q_\alpha \left\{ M\, S_\beta\, G_1(Q^2, \nu) + \frac{p.q}{M}\left[S_\beta - \frac{S.q}{p.q}\, p_\beta \right] G_2(Q^2,\nu) \right\}$$

G_1, G_2 : SPIN - DEPENDENT NUCLEON STRUCTURE FUNCTIONS

- **'SCALING' FUNCTIONS**

INSTEAD OF ν : USE $x_B \equiv \dfrac{Q^2}{2M\nu}$ BJORKEN SCALING VARIABLE

$$M W_1 \equiv F_1(Q^2, x_B)$$
$$\nu W_2 \equiv F_2(Q^2, x_B)$$
$$M^2 \nu G_1 \equiv g_1(Q^2, x_B)$$
$$M \nu^2 G_2 \equiv g_2(Q^2, x_B)$$

↳ F_1, F_2, g_1, g_2 ARE DIMENSIONLESS

$$W_S^{\mu\nu} = 2\left\{\left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) F_1 + \frac{1}{p\cdot q}\left(p^\mu - \frac{p\cdot q}{q^2} q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2} q^\nu\right) F_2\right\}$$

$$W_A^{\mu\nu} = 2\,\varepsilon^{\mu\nu\alpha\beta} q_\alpha \frac{1}{\nu}\left\{S_\beta\, g_1 + \left(S_\beta - \frac{S\cdot q}{p\cdot q} p_\beta\right) g_2\right\}$$

- UNPOLARIZED CROSS SECTION

$$\left(\frac{d\sigma}{d\Omega_e' \, dE_e'}\right)^{LAB} = \frac{\alpha^2}{Q^4} \cdot \frac{E_e'}{E_e} \cdot \frac{1}{2M} \cdot L^S_{\mu\nu} \cdot W_S^{\mu\nu}$$

$$\Rightarrow L^S_{\mu\nu} \cdot H_S^{\mu\nu} = 2 \left\{ k_\mu k'_\nu + k_\nu k'_\mu - k.k' \, g_{\mu\nu} \right\}$$

$$\cdot \, 2 \left\{ \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) F_1 + \frac{1}{P.q} \left(P^\mu - \frac{P.q}{q^2} q^\mu\right)\left(P^\nu - \frac{P.q}{q^2} q^\nu\right) F_2 \right\}$$

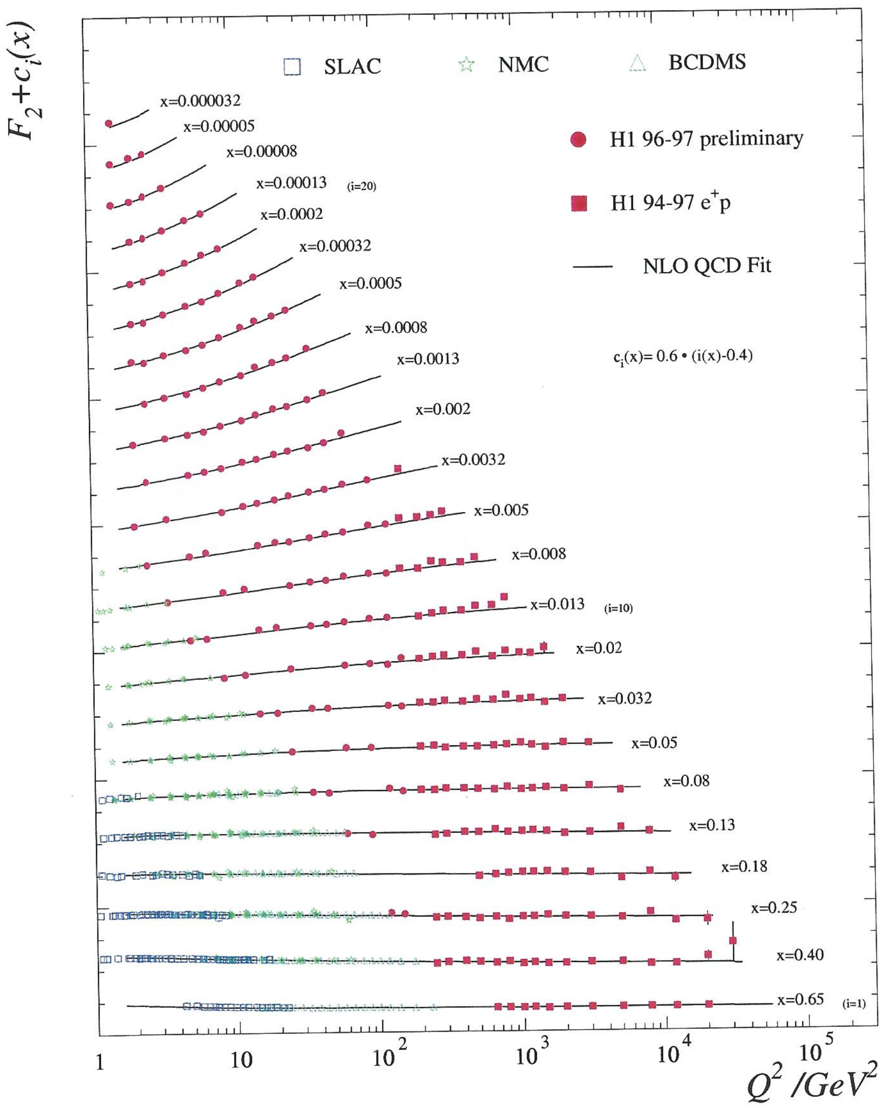
$$= 4 \left\{ 2\, k.k' \, F_1 + \frac{1}{P.q} \Big(2 (P.k)(P.k') - k.k' \, M^2 \Big) F_2 \right\}$$

$$= 4 \left\{ Q^2 F_1 + \frac{M^2}{P.q} E_e E_e' (1 + \cos\theta) \, F_2 \right\}$$

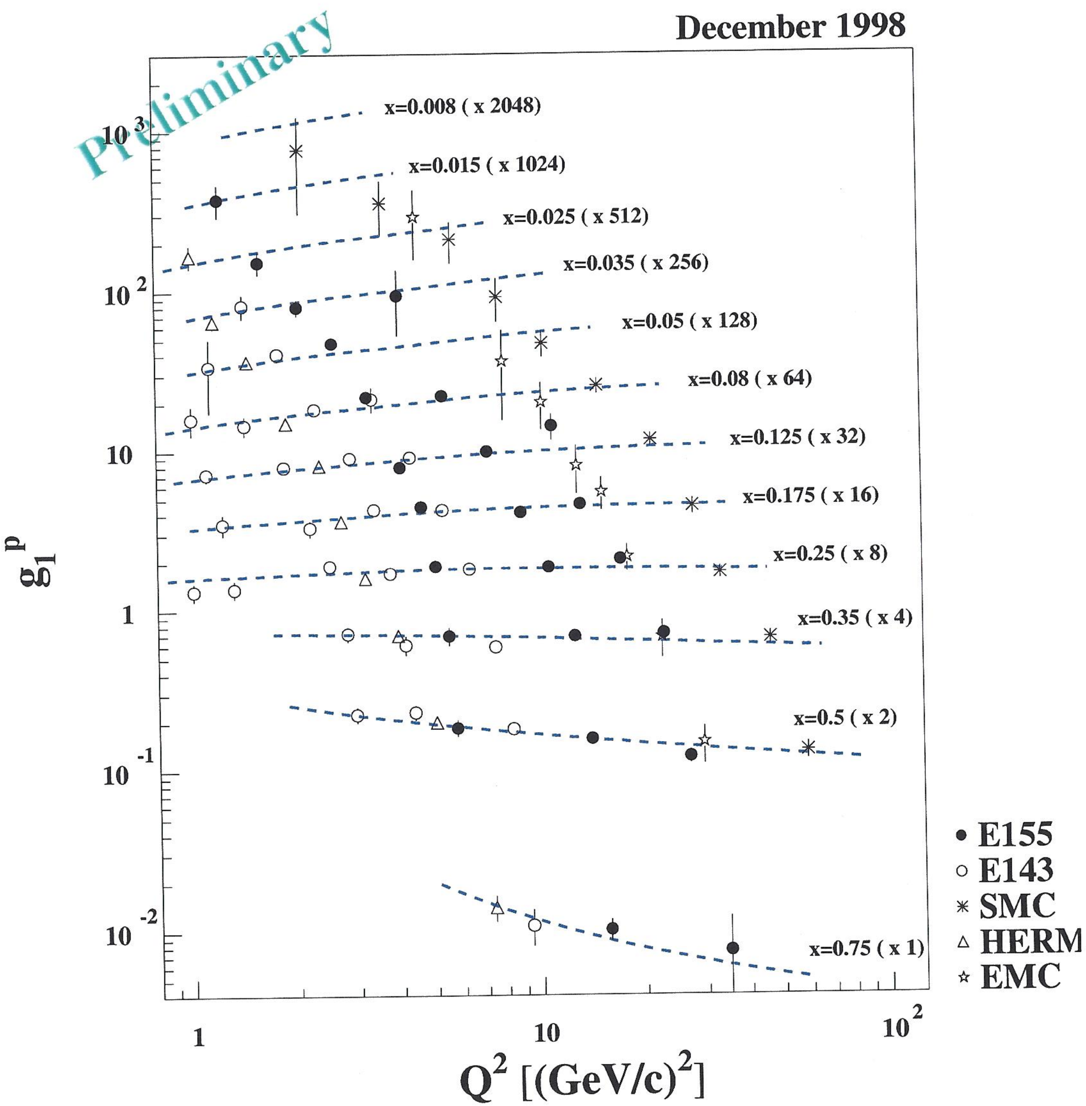
$$= 8 \, E_e E_e' \left\{ 2 \sin^2\theta/2 \; F_1 + \frac{M}{\nu} \cos^2\theta/2 \; F_2 \right\}$$

$$\left(\frac{d\sigma}{d\Omega_e' \, dE_e'}\right)^{LAB}_{UNPOL} = \frac{\alpha^2}{Q^4} \, \frac{4 E_e'^2}{M} \left\{ 2 \sin^2\theta/2 \; F_1 + \frac{M}{\nu} \cos^2\theta/2 \; F_2 \right\}$$

World data on F_1^p



World data on g_1^p



BJORKEN SCALING

IN LIMIT $Q^2 \gg$

$\nu \gg$

$$x_B = \frac{Q^2}{2M\nu} = \text{CONSTANT}$$

$$F_1(x_B, Q^2) \longrightarrow F_1(x_B)$$

$$F_2(x_B, Q^2) \longrightarrow F_2(x_B)$$

AND } $F_2 = 2x_B F_1$

(CALLAN - GROSS RELATION)

$$g_1(x_B, Q^2) \longrightarrow g_1(x_B)$$

$$g_2(x_B, Q^2) \longrightarrow g_2(x_B)$$

⇒ HADRONIC TENSOR IN PARTON MODEL

- $$W^{\mu\nu} = \frac{1}{2\pi} \sum_X (2\pi)^4 \delta^4(q+p-p_X) \langle N | J^{\mu +}(0) | X \rangle \langle X | J^{\nu}(0) | N \rangle$$

IN PARTON MODEL

$Q^2 \gg$

$\nu \gg$

$x_B = \frac{Q^2}{2M\nu}$ = CONSTANT

EVALUATE TENSOR IN FRAME WHERE PROTON MOVES WITH LARGE MOMENTUM (INFINITE MOMENTUM FRAME)

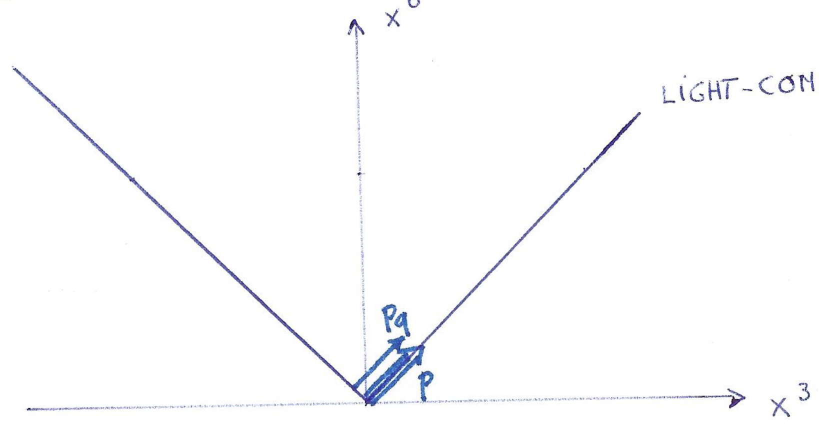
↳ THE QUARKS MOVE NEARLY COLLINEAR WITH PROTON (i.e. NEGLECT THEIR SMALL TRANSVERSE MOMENTUM $\vec{p}_{q\perp}$)

$$p_q \approx x\, p$$

$$\langle \vec{p}_{q\perp}^{\,2} \rangle \approx (0.3 \text{ GeV})^2 \ll Q^2$$

$p_q^2 \approx 0$ (QUARKS ARE NEARLY MASSLESS)

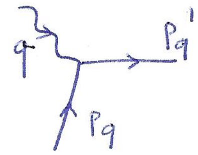
SPACE-TIME PICTURE



IN PARTON MODEL (INCOHERENT SCATTERING OFF INDIVIDUAL QUARKS)

$$W^{\mu\nu}_{\text{PARTON MODEL}} = \sum_q \sum_s \int_0^1 \frac{dx}{x} \; \underbrace{n_q(x,s)}_{} \cdot \underbrace{e_q^2 \, w^{\mu\nu}}_{}$$

- $\sum_q$ ↑ SUM OVER QUARK SPECIES (FLAVORS)
- $\sum_s$ ↑ SUM OVER INITIAL QUARK SPIN PROJECTIONS
- $\frac{dx}{x}$ ↑ INITIAL FLUX $E_q = x E_N$
- $n_q(x,s)$ ↓ NUMBER DENSITY OF QUARKS WITH MOMENTUM FRACTION x & SPIN PROJECTION $s = \pm \frac{1}{2}$
- $e_q^2 w^{\mu\nu}$ ↓ SCATTERING OFF A SINGLE QUARK



- SCATTERING OFF AN INDIVIDUAL QUARK : $w^{\mu\nu}$

$$w^{\mu\nu} = \frac{1}{2\pi} \underbrace{\sum_{s'} \int \frac{d^3 \vec{P}_q'}{(2\pi)^3 \, 2E_q'}}_{\sum_X} (2\pi)^4 \delta^4(q + P_q - P_q')$$

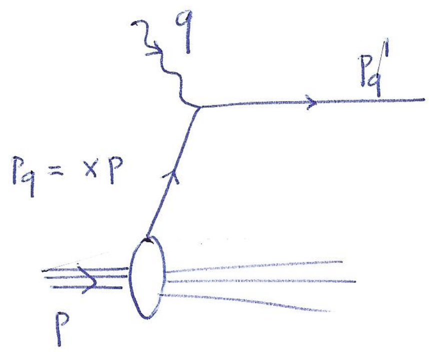
$$\cdot \, \bar{u}(P_q\, s) \, \gamma^\mu \, u(P_q'\, s') \, \bar{u}(P_q'\, s') \, \gamma^\nu \, u(P_q\, s)$$

$$= \underbrace{\frac{1}{2E_q'} \delta(P_q'^0 - E_q')}_{\substack{||| \\ \delta(P_q'^2 - m_q^2) \\ m_q^2 \simeq 0}} \quad (P_q'^0 > 0) \quad \cdot \quad \mathrm{Tr}\left\{ \left(\frac{1 + (2s)\gamma_5}{2} \right) \not{P}_q \, \gamma^\mu \, \not{P}_q' \, \gamma^\nu \right\}$$

↓ $P_q = xP$

$$\delta(P_q'^2) = \delta\left((q + xP)^2\right) = \delta(x \, 2P.q - Q^2)$$

$$= \frac{1}{2P.q} \, \delta\left(x - \frac{Q^2}{2P.q}\right)$$



$$\delta(p_q'^2) = \frac{1}{Q^2}\left(\frac{Q^2}{2P\cdot q}\right)\delta\left(x - \frac{Q^2}{2P\cdot q}\right)$$

$\Downarrow$ USING $x_B \equiv \frac{Q^2}{2P\cdot q}$ $\left(= \frac{Q^2}{2M\nu} \text{ IN LAB SYSTEM} \right)$

$$\delta(p_q'^2) = \frac{x_B}{Q^2}\,\delta(x - x_B)$$

$\Downarrow$

PHOTON SCATTERS OFF QUARK WHICH HAS MOMENTUM FRACTION x_B OF PROTON MOMENTUM

$$\therefore \quad w^{\mu\nu} = \frac{x_B}{Q^2}\,\delta(x-x_B) \cdot \frac{1}{2}\left\{ \mathrm{Tr}\{\not p_q \gamma^\mu \not p_q' \gamma^\nu\} + (2s)\,\mathrm{Tr}\{\gamma_5 \not p_q \gamma^\mu \not p_q' \gamma^\nu\} \right\}$$

$$w^{\mu\nu} = \frac{x_B}{Q^2}\,\delta(x-x_B) \cdot 2\left\{ p_q^\mu p_q'^\nu + p_q^\nu p_q'^\mu - (p_q\cdot p_q')\,g^{\mu\nu} + (2s)\, i\, \varepsilon^{\mu\nu\alpha\beta} p'_{q\alpha} p_{q\beta} \right\}$$

$$\downarrow \qquad p_q = x\,P$$

$$p_q' = q + x\,P$$

$$w^{\mu\nu} = \frac{x_B}{Q^2}\,\delta(x - x_B)$$

$$\cdot 2 \cdot \left\{ x_B^2 \left[P^\mu \left(P^\nu + \frac{1}{x_B} q^\nu \right) + P^\nu \left(P^\mu + \frac{1}{x_B} q^\mu \right) \right] \right.$$

$$\left. - \frac{Q^2}{2} g^{\mu\nu} + (2s)\, i\, \varepsilon^{\mu\nu\alpha\beta}\, x_B\, q_\alpha P_\beta \right\}$$

$$\downarrow$$

$$w^{\mu\nu} = \frac{x_B}{Q^2}\,\delta(x - x_B)$$

$$\cdot 2 \cdot \left\{ 2 x_B^2 \left(P^\mu - \frac{P \cdot q}{q^2} q^\mu \right) \left(P^\nu - \frac{P \cdot q}{q^2} q^\nu \right) \right.$$

$$+ \frac{Q^2}{2} \left(- g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right)$$

$$\left. + x_B\, (2s)\, i\, \varepsilon^{\mu\nu\alpha\beta}\, q_\alpha P_\beta \right\}$$

↳
$$W^{\mu\nu}_{\substack{\text{PARTON}\\ \text{MODEL}}} = \sum_q e_q^2 \sum_s n_q(x_B,s) \cdot \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right)$$
$$+ \sum_q e_q^2 \sum_s n_q(x_B,s) \cdot \frac{4x_B^2}{Q^2} \left(p^\mu - \frac{p\cdot q}{q^2} q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2} q^\nu\right)$$
$$+ \sum_q e_q^2 \sum_s (2s)\, n_q(x_B,s) \cdot \frac{2x_B}{Q^2}\, i\, \varepsilon^{\mu\nu\alpha\beta} q_\alpha P_\beta$$

COMPARE THIS WITH GENERAL EXPRESSION

$$W^{\mu\nu} = 2 \left\{ \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) F_1 \right.$$
$$+ \frac{1}{p\cdot q}\left(p^\mu - \frac{p\cdot q}{q^2} q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2} q^\nu\right) F_2$$
$$\left. + i\, \varepsilon^{\mu\nu\alpha\beta} q_\alpha \frac{M}{p\cdot q}\left[S_\beta\, g_1 + S_{\perp\beta}\, g_2\right] \right\}$$

- UNPOLARIZED STRUCTURE FUNCTIONS

IN PARTON MODEL

$$F_1(x_B, Q^2) = \frac{1}{2} \sum_q e_q^2 \sum_s n_q(x_B,s)$$ FUNCTION OF x_B ONLY

$$F_2(x_B, Q^2) = x_B \sum_q e_q^2 \sum_s n_q(x_B,s)$$ FUNCTION OF x_B ONLY!

⇓

BJORKEN SCALING

$$\sum_s n_q(x_B, s) = n_q(x_B, +\tfrac{1}{2}) + n_q(x_B, -\tfrac{1}{2})$$

$$\equiv q(x_B)$$ UNPOLARIZED QUARK DISTRIBUTION IN NUCLEON

FOR EACH FLAVOR

$u(x_B),\ d(x_B),\ s(x_B),\ \dots$ QUARK NUMBER DENSITIES IN NUCLEON

IN PARTON MODEL

$$F_1(x_B, Q^2) = \frac{1}{2}\sum_q e_q^2 \left\{ q(x_B) + \bar{q}(x_B) \right\}$$

↳ ANTI-QUARK CONTRIBUTION

$$= \frac{1}{2}\left\{ \frac{4}{9}u(x_B) + \frac{1}{9}d(x_B) + \frac{1}{9}s(x_B) + \dots + \frac{4}{9}\bar{u}(x_B) + \frac{1}{9}\bar{d}(x_B) + \frac{1}{9}\bar{s}(x_B) + \dots \right\}$$

$$F_2(x_B, Q^2) = \sum_q e_q^2\, x_B\left\{ q(x_B) + \bar{q}(x_B) \right\}$$

↳ QUARK MOMENTUM DENSITIES

$$F_2(x_B, Q^2) = 2x_B\, F_1(x_B, Q^2)$$

↑ IN PARTON MODEL

: CALLAN - GROSS RELATION IS A CONSEQUENCE OF SPIN 1/2 NATURE OF PARTONS

↓

QUARKS ARE DIRAC PARTICLES

- SPIN DEPENDENT STRUCTURE FUNCTIONS

NUCLEON MOVING WITH HIGH VELOCITY

↳ SPIN ALIGNED ALONG MOMENTUM (POSITIVE HELICITY)

$$S^\beta \approx \frac{p^\beta}{M} \qquad S.p = M \approx 0 \quad (E_N \gg M)$$

$$S_\perp^\beta \approx 0 \Rightarrow$$ IN PARTON MODEL $g_2(x_B, Q^2)$ CANNOT BE ACCESSED

$$g_1(x_B, Q^2) = \frac{1}{2} \sum_q e_q^2 \sum_s (2s)\, n_q(x_B, s)$$

$$\sum_s (2s)\, n_q(x_B, s) = n_q(x_B, +\tfrac{1}{2}) - n_q(x_B, -\tfrac{1}{2})$$

$$\equiv \Delta q(x_B)$$ QUARK HELICITY DISTRIBUTION IN NUCLEON

$$\boxed{g_1(x_B, Q^2) = \frac{1}{2} \sum_q e_q^2 \left\{ \Delta q(x_B) + \Delta \bar{q}(x_B) \right\}}$$ BJORKEN SCALING

$$= \frac{1}{2} \left\{ \frac{4}{9} \Delta u(x_B) + \frac{1}{9} \Delta d(x_B) + \frac{1}{9} \Delta s(x_B) + \dots \right.$$

$$\left. + \frac{4}{9} \Delta \bar{u}(x_B) + \frac{1}{9} \Delta \bar{d}(x_B) + \frac{1}{9} \Delta \bar{s}(x_B) + \dots \right\}$$

⇒ VALENCE & SEA-QUARK DISTRIBUTIONS

$q(x_B)$: QUARK DISTRIBUTION IN PROTON

$\bar{q}(x_B)$: ANTI-QUARK DISTRIBUTION IN PROTON

- "VALENCE" DISTRIBUTION

$$q_V(x_B) \equiv q(x_B) - \bar{q}(x_B)$$

- "SEA" DISTRIBUTION

$$q_S(x_B) \equiv \bar{q}(x_B)$$

$$\rightsquigarrow \boxed{q(x_B) = q_V(x_B) + q_S(x_B)}$$

⇒ UNPOLARIZED SUM RULES

- PROTON : $|p\rangle = c_1 |uud\rangle + c_2 |uud\,u\bar{u}\rangle + c_3 |uud\,d\bar{d}\rangle + \ldots$

$$\int_0^1 dx \; u_V(x) = 2$$

$$\int_0^1 dx \; d_V(x) = 1$$

$$\int_0^1 dx \left[s(x) - \bar{s}(x) \right] = 0$$

NET STRANGENESS OF PROTON $= 0$

- NEUTRON:

q^p: QUARK DISTR IN PROTON

q^n: QUARK DISTR IN NEUTRON

$$|n\rangle = c_1\,|ddu\rangle + c_2\,|ddu\,d\bar{d}\rangle + c_3\,|ddu\,u\bar{u}\rangle + \dots$$

↳ OBTAINED FROM $|p\rangle$ BY $u \leftrightarrow d$

$$u^n(x) = d^p(x) \equiv d(x)$$

$$d^n(x) = u^p(x) \equiv u(x)$$

SU(2) SYMMETRY

$$\int_0^1 dx\; d_V^n(x) = \int_0^1 dx\; u_V(x) = 2$$

$$\int_0^1 dx\; u_V^n(x) = \int_0^1 dx\; d_V(x) = 1$$

$$F_1^p = \frac{1}{2}\left\{\frac{4}{9}(u+\bar{u}) + \frac{1}{9}(d+\bar{d}) + \frac{1}{9}(s+\bar{s})\right\}$$

$$F_1^n = \frac{1}{2}\left\{\frac{4}{9}(d+\bar{d}) + \frac{1}{9}(u+\bar{u}) + \frac{1}{9}(s+\bar{s})\right\}$$

∴ BY DOING EXPERIMENTS ON BOTH PROTON & NEUTRON

$\Longrightarrow$ u, d QUARK FLAVOR SEPARATION

⇒ SEPARATION OF QUARK & ANTI-QUARK DISTRIBUTIONS

HOW CAN WE SEPARATE q & $\bar{q}$?

↳ EM PROBE COUPLES IN SAME WAY TO q & $\bar{q}$

$$F_1 = \frac{1}{2} e_q^2 (q + \bar{q})$$

↳ NEED PROBE WHICH COUPLES DIFFERENTLY TO q & $\bar{q}$

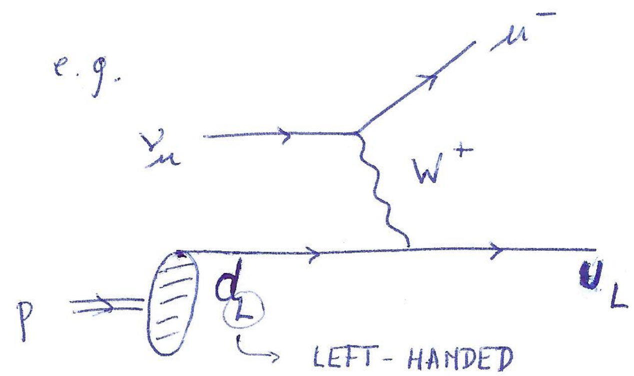
MASSLESS q : LEFT-HANDED

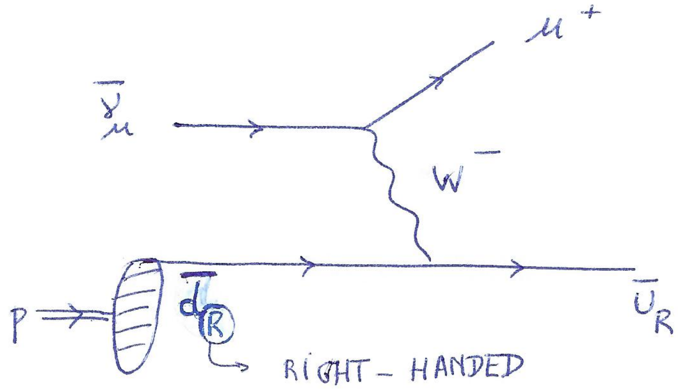
MASSLESS $\bar{q}$: RIGHT-HANDED

⇓

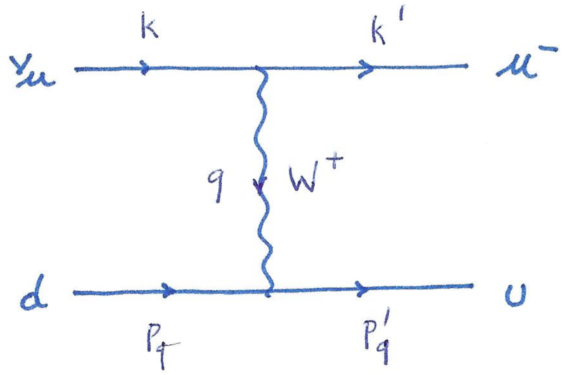
WEAK INTERACTION COUPLES DIFFERENTLY TO q & $\bar{q}$

e.g.





⇒ NEUTRINO – QUARK SCATTERING



$$(-i)\,\frac{g_W}{2\sqrt{2}}\,\gamma_\alpha(1-\gamma_5)$$

$$-\frac{i\,g^{\alpha\beta}}{q^2-m_W^2}$$

$$(-i)\,\frac{g_W}{2\sqrt{2}}\,\gamma_\beta(1-\gamma_5)$$

(NEGLECT $d \leftrightarrow s$ MIXING)

WITH

$$\boxed{\frac{g_W^2}{8m_W^2} \equiv \frac{G_F}{\sqrt{2}}}$$

→ FERMI WEAK COUPLING CONSTANT

FOR $-q^2 \ll m_W^2$ → NEGLECT q^2 IN W PROPAGATOR

- $$\mathcal{M} = -i\left(\frac{G_F}{\sqrt{2}}\right)\cdot \bar{u}_{\mu^-}\gamma_\nu(1-\gamma_5)\,u_{\nu_\mu}\cdot \bar{u}_u\,\gamma^\nu(1-\gamma_5)\,u_d$$

- CROSS SECTION

$$d\sigma = \frac{1}{2E_k\,2E_{p_q}\,v_{rel}}\;\frac{d^3\bar{k}'}{(2\pi)^3\,2E_k'}\;\frac{d^3\bar{p}_q'}{(2\pi)^3\,2E_{p_q}'}\;(2\pi)^4\,\delta^4(k+p_q-k'-p_q')\cdot|\mathcal{M}|^2$$

↓ NEGLECT MASSES OF ALL PARTICLES

+ CONSIDER c.m. SYSTEM $\quad |\bar{k}| = |\bar{p}_q| = |\bar{k}'| = |\bar{p}_q'| = \frac{\sqrt{\hat{s}}}{2}$

$$\left.\begin{aligned}\hat{s} &= (k+p_q)^2\\ \hat{u} &= (k'-p_q)^2\\ \hat{t} &= (k-k')^2 = q^2 = -Q^2\end{aligned}\right\}\quad \hat{s}+\hat{u} = Q^2$$

$\hookrightarrow \quad 2E_k \; 2E_{p_q} \; v_{rel} = 4E_k E_{p_q} \left(\frac{|\bar{k}|}{E_k} + \frac{|\bar{k}|}{E_{p_q}} \right)$

$= 4|\bar{k}|\sqrt{\hat{s}} = 2\hat{s}$

$\hookrightarrow \quad \hat{t} = -2k.k' = -2|\bar{k}|^2(1-\cos\Theta_{cm})$

$d\hat{t} = +2|\bar{k}|^2 \, d\cos\Theta_{cm}$

$\hookrightarrow \quad \int d\phi \rightarrow 2\pi$ $\qquad \hookrightarrow \; \delta(\sqrt{\hat{s}} - 2|\bar{k}'|) = \frac{1}{2}\delta(|\bar{k}'| - \frac{\sqrt{\hat{s}}}{2})$

$$\frac{d\sigma}{d\hat{t}} = \frac{1}{2\hat{s}} \cdot \frac{1}{2\pi} \cdot \frac{1}{2\hat{s}} \cdot \frac{1}{2} \cdot |\mathcal{M}|^2$$

$\frac{1}{2\hat{s}}$ ← $\frac{1}{4E_k E_{p_q} v_{rel}}$

$\frac{1}{2\pi}$: FROM $\int d\phi$

$\frac{1}{2\hat{s}}$ → $\frac{1}{2 \cdot 4E_{k'} E_{p_q'}}$

$\frac{1}{2}$: FROM δ

$$\boxed{\frac{d\sigma}{d\hat{t}} = \frac{1}{16\pi\hat{s}^2} \cdot \left(\frac{G_F}{\sqrt{2}}\right)^2 \cdot L_{\mu\nu} H^{\mu\nu}}$$

WITH

→ NO POLARIZATION AVERAGE FOR ν_μ (LEFT-HANDED)

$$L_{\mu\nu} = Tr\left\{ \gamma_\mu(1-\gamma_5) \not{k}' \gamma_\nu (1-\gamma_5) \not{k} \right\}$$

$$H^{\mu\nu} = \frac{1}{2} Tr\left\{ \gamma^\mu (1-\gamma_5) \not{p}_q' \gamma^\nu (1-\gamma_5) \not{p}_q \right\}$$

↑ $\frac{1}{2}$: AVERAGE OVER POLARIZATION OF INITIAL QUARK

- $$L_{\mu\nu} = 2\,\mathrm{Tr}\left\{ \gamma_\mu \not{k}' \gamma_\nu \not{k} \right\} + 2\,\mathrm{Tr}\left\{ \gamma_5 \gamma_\mu \not{k}' \gamma_\nu \not{k} \right\}$$
$$= 8 \left\{ k_\mu k'_\nu + k_\nu k'_\mu - k.k' g_{\mu\nu} + i\,\varepsilon_{\mu\nu\alpha\beta}\, k^\alpha k'^\beta \right\}$$

↳ SAME AS LEPTON TENSOR FOR e^- SCATTERING
WHEN PUTTING $(2h) = -1$ (ν ARE LEFT-HANDED)

- $$H^{\mu\nu} = \frac{8}{2} \left\{ P_q^\mu P_q'^\nu + P_q^\nu P_q'^\mu - P_q.P_q' g^{\mu\nu} + i\,\varepsilon^{\mu\nu\gamma\delta} (P_q)_\gamma (P_q')_\delta \right\}$$

- $$L_{\mu\nu} H^{\mu\nu} = 32 \left\{ k_\mu k'_\nu + k_\nu k'_\mu - \frac{Q^2}{2} g_{\mu\nu} \right\} \left\{ 2 P_q^\mu P_q^\nu - \frac{Q^2}{2} g^{\mu\nu} \right\}$$
$$- 32 \underbrace{\varepsilon_{\mu\nu\alpha\beta}\, \varepsilon^{\mu\nu\gamma\delta}}_{-2\left(g_\alpha^\gamma g_\beta^\delta - g_\alpha^\delta g_\beta^\gamma\right)} k^\alpha k'^\beta (P_q)_\gamma (P_q')_\delta$$

$$= 32 \left\{ 4 (P_q . k)(P_q . k') + \frac{1}{2} (Q^2)^2 \right.$$
$$\left. + 2 (P_q . k)(P_q' . k') - 2 (P_q . k')(P_q' . k) \right\}$$

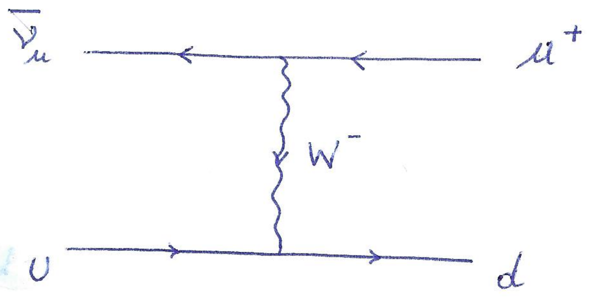
$$= 32 \left\{ - \hat{s}\hat{u} + \frac{1}{2} (\hat{s} + \hat{u})^2 \right.$$
$$\left. + \frac{1}{2} \hat{s}^2 - \frac{1}{2} \hat{u}^2 \right\}$$

$$= 32\, \hat{s}^2$$

$$\hookrightarrow \boxed{\frac{d\sigma}{d\hat{t}}(\nu_\mu d \to \mu^- u) = \frac{G_F^2}{32\pi\hat{s}^2} \cdot 32\,\hat{s}^2 = \frac{G_F^2}{\pi}}$$

$$= \frac{d\sigma}{d\hat{t}}(\bar{\nu}_\mu \bar{d} \to \mu^+ \bar{u})$$

$\hookrightarrow$ ANALOGOUSLY $\bar{\nu}_\mu u \to \mu^+ d$



ANTI-ν IS RIGHT-HANDED

W^- COUPLES ONLY TO LEFT-HANDED QUARKS

$$L_{\mu\nu} = Tr\{\gamma_\nu(1-\gamma_5)k'\gamma_\mu(1-\gamma_5)k\}$$

$\hookrightarrow$ $\mu \leftrightarrow \nu$ COMPARED TO $L_{\mu\nu}$ FOR $\nu_\mu d \to \mu^- u$

$H^{\mu\nu}$ SAME AS FOR $\nu_\mu d \to \mu^- u$

$$\therefore \; L_{\mu\nu}H^{\mu\nu} = 32\left\{-\hat{s}\hat{u} + \frac{1}{2}(\hat{s}+\hat{u})^2 \ominus \left[\frac{1}{2}\hat{s}^2 - \frac{1}{2}\hat{u}^2\right]\right\} = 32\,\hat{u}^2$$

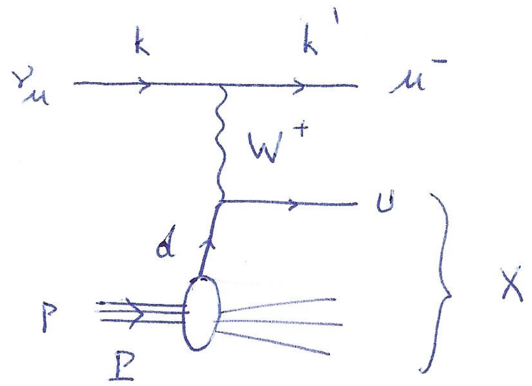
$\uparrow$ DIFFERENT RELATIVE SIGN

$$\boxed{\frac{d\sigma}{d\hat{t}}(\bar{\nu}_\mu u \to \mu^+ d) = \frac{G_F^2}{\pi}\cdot\frac{\hat{u}^2}{\hat{s}^2}}$$

$$= \frac{d\sigma}{d\hat{t}}(\nu_\mu \bar{u} \to \mu^- \bar{d})$$

⇒ NEUTRINO – PROTON SCATTERING

(NEGLECT ALL MASSES)



$$t = k - k'^2$$

$$s = (k + P)^2$$

$$\frac{d\sigma}{dt}(\nu p \to \mu^- X) = \sum_q \int_0^1 dx \; q(x) \cdot \underbrace{\frac{d\sigma}{dt}(\nu q \to \mu^- q')}$$

WITH $\underline{\underline{\hat{s} = x s}}$

↑ PARTONIC

$$\boxed{\frac{d\sigma}{dt\, dx_B}(\nu p \to \mu^- X) = \sum_q q(x_B) \cdot \frac{d\sigma}{dt}(\nu q \to \mu^- q')\Big|_{\hat{s} = x_B s}}$$

↓ INTRODUCE DIMENSIONLESS VARIABLE

$$y \equiv \frac{q \cdot P}{k \cdot P} = \frac{2\, q \cdot P}{s} = \frac{Q^2}{s\, x_B}$$

$$\underline{\underline{Q^2 = s\, x_B\, y}} \qquad dQ^2 = s\, x_B\, dy$$

$$y = \frac{q \cdot (x P)}{k \cdot (x P)} = \frac{(k - k') \cdot P_q}{k \cdot P_q} = \frac{\hat{s} + \hat{u}}{\hat{s}}$$

↑ PARTONIC VARIABLES

$$\frac{\hat{u}}{\hat{s}} = -(1-y)$$

$$\Downarrow$$

$$\frac{d\sigma}{d\hat{t}}(\bar{\nu} u \to \mu^+ d) = \frac{G_F^2}{\pi}(1-y)^2$$

$$\frac{d\sigma}{dx_B\,dy}(\nu p \to \mu^- X)$$

$$= s\,x_B \cdot \frac{d\sigma}{dt\,dx_B}(\nu p \to \mu^- X)$$

$$= s\,x_B \cdot \frac{G_F^2}{\pi}\left\{ d(x_B) + \bar{u}(x_B)\cdot(1-y)^2 \right\}$$

$$\Rightarrow \quad \frac{d\sigma}{dx_B\,dy}(\nu p \to \mu^- X) = \frac{G_F^2 s}{\pi}\left\{ x_B\, d(x_B) + x_B\,\bar{u}(x_B)(1-y)^2 \right\}$$

$$\Rightarrow \quad \frac{d\sigma}{dx_B\,dy}(\bar{\nu} p \to \mu^+ X) = \frac{G_F^2 s}{\pi}\left\{ x_B\, u(x_B)(1-y)^2 + x_B\,\bar{d}(x_B) \right\}$$

QUALITATIVE DIFFERENCE $\quad \nu$ CROSS SECTION $\sim d(x_B)$

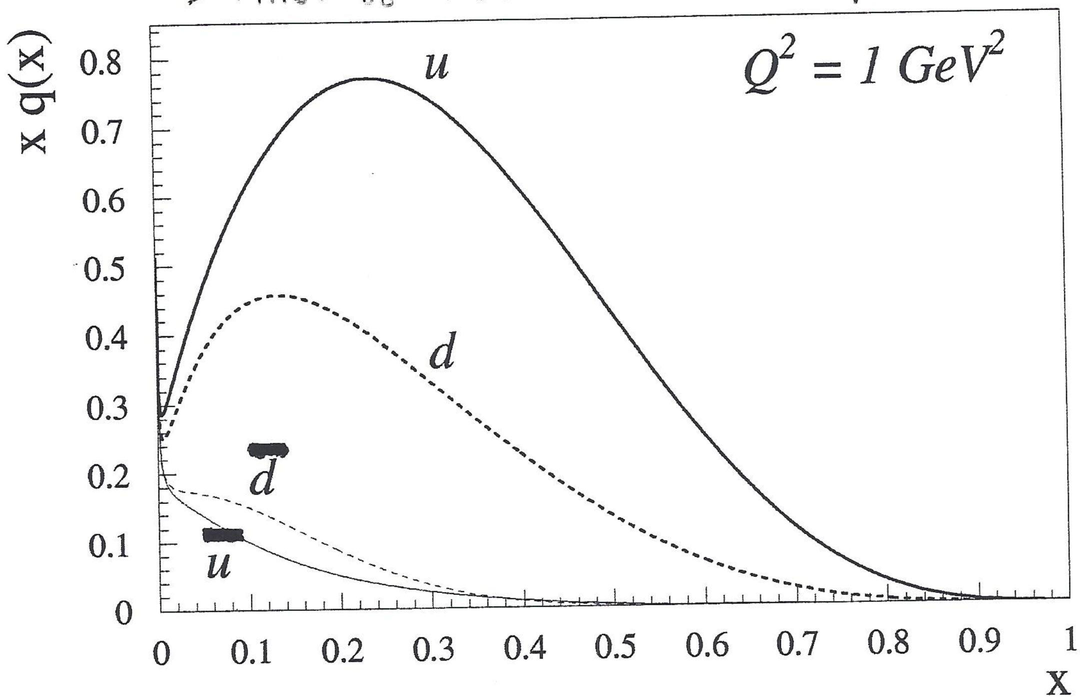
$\bar{\nu}$ CROSS SECTION $\sim u(x_B)(1-y)^2$

& VICE VERSA FOR $\bar{q}$

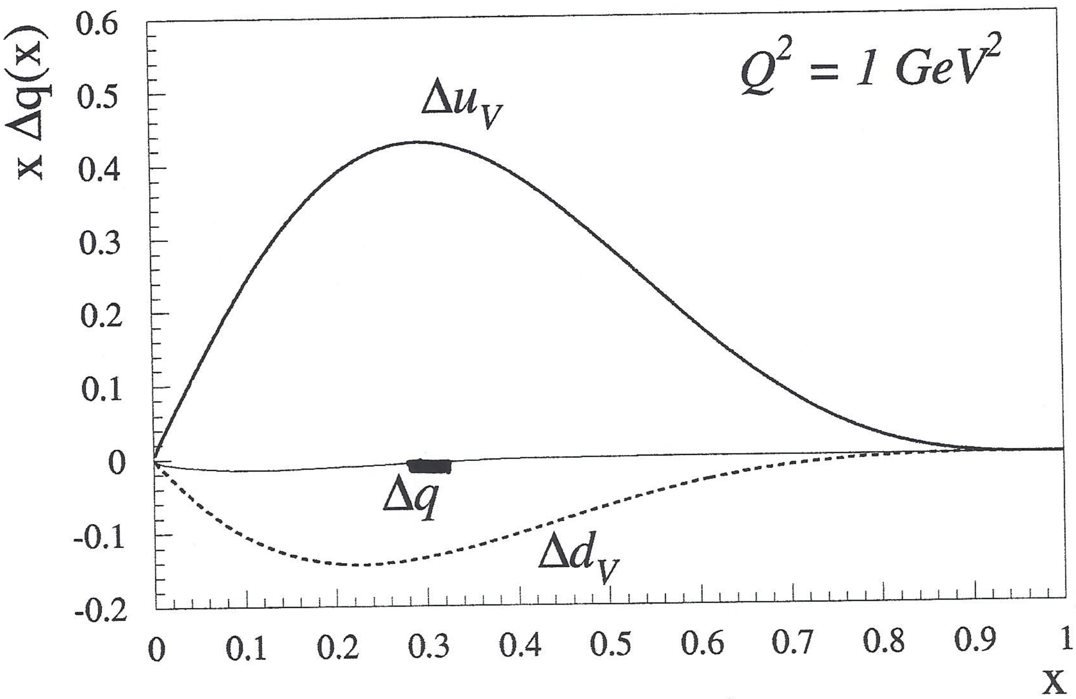
QUARK DISTRIBUTIONS in the PROTON

$q(x) = q_V(x) + q_S(x)$ $\qquad$ $q_S(x) = \bar{q}(x)$

⇒ MRST 98 NLO QUARK DISTR. $q(x)$



⇒ LEADER, SIDOROV, STAMENOV 98 NLO $\Delta q(x)$



UNPOLARIZED SUM RULES

$$q(x) = q_v(x) + q_s(x)$$
$$\bar{q}(x) = q_s(x)$$

⇒ **SUM RULES FOR 1 FLAVOR** (PROTON)

$$\int_0^1 dx\, u_v(x) = 2$$

$$\int_0^1 dx\, d_v(x) = 1$$

$$\int_0^1 dx\, [s(x) - \bar{s}(x)] = 0$$

$$|p\rangle = c_1 |uud\rangle + c_2 |uud u\bar{u}\rangle + c_3 |uud d\bar{d}\rangle + \ldots$$

⇒ **GROSS - LLEWELLYN - SMITH SUM RULE** (# VALENCE QUARK

$$S_{GLS} \equiv \frac{1}{2} \int_0^1 dx \left[F_3^{\nu p}(x, Q^2) + F_3^{\bar{\nu} p}(x, Q^2) \right]$$

$$= \int_0^1 dx \left[u(x,Q^2) - \bar{u}(x,Q^2) + d(x,Q^2) - \bar{d}(x,Q \right.$$

$$= 3 \left[1 - \frac{\alpha_s(Q^2)}{\pi} + O(\alpha_s^2) \right]$$

(CCFR) EXP. $S_{GLS}(Q^2 = 3\,GeV^2) = 2.50 \pm 0.018\,(stat) \pm 0.078\,(s$

THEORY $S_{GLS}(Q^2 = 3\,GeV^2) = 2.66 \pm 0.04$ ✓

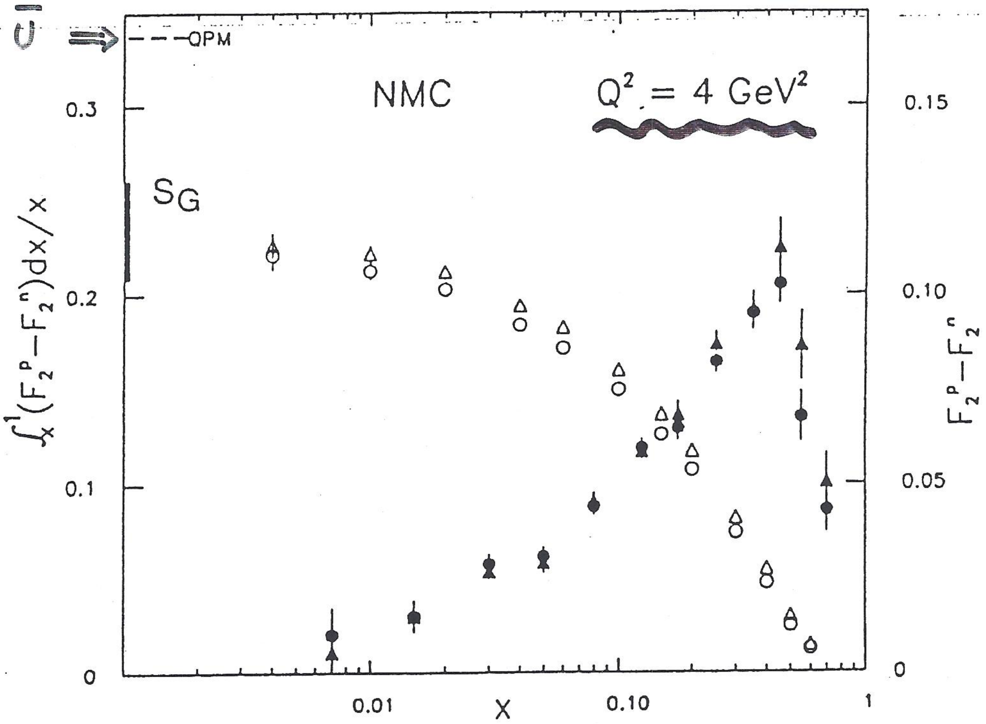
GOTTFRIED SUM RULE

$$\Rightarrow \quad F_2^p - F_2^n = \frac{x}{3}\left[u(x) + \bar{u}(x) - d(x) - \bar{d}(x)\right]$$

$$= \frac{x}{3}\left[u_v(x) - d_v(x)\right] + \frac{2x}{3}\left[\bar{u}(x) - \bar{d}(x)\right]$$

$$S_G \equiv \int_0^1 dx\, \frac{1}{x}\left(F_2^p - F_2^n\right) = \frac{1}{3} - \frac{2}{3}\int_0^1 dx\left[\bar{d}(x) - \bar{u}(x)\right]$$

if $\bar{d} = \bar{u} \Rightarrow$



(NMC 97) $\quad S_G = 0.2281 \pm 0.0065\ (\text{stat})$

$$\Downarrow \qquad \bar{d} > \bar{u}$$

$$\int_0^1 dx\left(\bar{d} - \bar{u}\right) \simeq 0.15$$

MOMENTUM SUM RULE

$$M_2^q(Q^2) \equiv \int_0^1 dx \; x \left[q(x,Q^2) + \bar{q}(x,Q^2) \right]$$

↳ MOMENTUM FRACTION OF PROTON CARRIED BY QUARK OF FLAVOR q

⇒ M_2^q AT LOW SCALE : $Q^2 = 1\ \mathrm{GeV}^2$

⇒ MRST 98 NLO QUARK DISTR.

q	$M_2^q\ (Q^2 = 1\ \mathrm{GeV}^2)$
u	0.40
d	0.22
s	0.03
SUM	0.65

⇒ M_2^q IN LIMIT $Q^2 \to \infty$

$$M_2^q(Q^2 \to \infty) = \frac{3N_f}{16 + 3N_f} \underset{N_f = 3}{=} 0.36$$

(FIXED POINT SOLUTION OF RENORMALIZATION GROUP EQ.)

∴ **GLUONS** CARRY AN IMPORTANT FRACTION OF PROTON MOMENTUM

MRST98 UNPOLARIZED parton distributions