STANDARD MODEL OF ELECTROWEAK INTERACTIONS

⇒ GAUGE SYMMETRY GROUP U(1) OF ELECTROMAGNETIC INTERACTION

ψ : DIRAC SPINOR FIELD

$$\mathcal{L}_0 = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi$$

* REQUIRE THEORY TO BE INVARIANT UNDER LOCAL PHASE TRANSFORMATION U(1)

$$\boxed{\psi(x) \xrightarrow{U(1)} e^{ie\chi(x)} \cdot \psi(x)}$$

$\chi(x)$: SCALAR FUNCTION

$$\partial_\mu \psi \longrightarrow e^{ie\chi} \left\{ \partial_\mu \psi + ie(\partial_\mu \chi)\psi \right\}$$

$$\bar{\psi}\gamma^\mu(\partial_\mu \psi) \longrightarrow \bar{\psi}\gamma^\mu(\partial_\mu \psi) + \underbrace{ie\,\bar{\psi}\gamma^\mu(\partial_\mu \chi)\psi}_{\text{EXTRA TERM}}$$

U(1) GAUGE INV. REQUIRES INTRODUCTION OF A VECTOR (GAUGE) FIELD THAT COMPENSATES EXTRA TERM

REPLACE : $\partial^\mu \Rightarrow D^\mu = \partial^\mu + ie A^\mu$

COVARIANT DERIVATIVE

GAUGE FIELD

$$A^\mu \xrightarrow{U(1)} A^\mu - \partial^\mu \chi$$

$$D^\mu \psi \xrightarrow{U(1)} e^{ie\chi} \left(D^\mu \psi \right)$$

$$\bar{\psi} \gamma_\mu (D^\mu \psi) \longrightarrow \bar{\psi} \gamma_\mu (D^\mu \psi)$$ INVARIANT

$$\boxed{\mathcal{L}_0 + \mathcal{L}_{INT} = \bar{\psi} \left(i \gamma^\mu \partial_\mu - m - e \gamma^\mu A_\mu \right) \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}}$$

U(1) INVARIANT

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

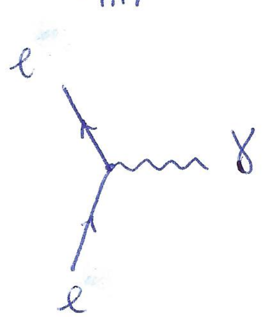
FIELD TENSOR

$$F^{\mu\nu} \xrightarrow{U(1)} F^{\mu\nu}$$

GAUGE FIELDS (γ)
MASSLESS

$$\mathcal{L}_{INT} = - e \, J^\mu_{e.m.} \, A_\mu$$

E. M. CURRENT



$$J^\mu_{e.m.} = \bar{\psi} \gamma^\mu \psi$$

$-ie\gamma^\mu$

FEYNMAN RULE

$\Rightarrow$ GAUGE SYMMETRY GROUP $SU(2) \times U(1)$ OF ELECTROWEAK INTERACTIONS

- $\boxed{\text{WEAK ISOSPIN : } SU(2)}$

↳ TAKE $\psi_{\nu_e}^L$, ψ_e^L TOGETHER (WEAK INTERACTIONS COUPLE BOTH)

WITH $\psi^L = \frac{1}{2}(1-\gamma_5)\psi$

$$\Psi_e^L = \begin{pmatrix} \psi_{\nu_e}^L \\ \psi_e^L \end{pmatrix}$$

↳ TAKE ψ_e^R SEPARATE (NO ν_e^R IN SM)

$$\boxed{\mathcal{L}_0 = \bar{\Psi}_e^L \, i\gamma^\mu \partial_\mu \Psi_e^L + \bar{\psi}_e^R \, i\gamma^\mu \partial_\mu \psi_e^R}$$

$$= \bar{\psi}_{\nu_e}^L \, i\gamma^\mu \partial_\mu \psi_{\nu_e}^L + \bar{\psi}_e^L \, i\gamma^\mu \partial_\mu \psi_e^L + \bar{\psi}_e^R \, i\gamma^\mu \partial_\mu \psi_e^R$$

↳ GAUGE SYMMETRY: GAUGE GROUP $SU(2)$ WEAK ISOSPIN (t, t_3)

	t	t_3	
ν_e^L	1/2	+1/2	$\Leftarrow$ WEAK ISOSPIN DOUBLET
e^L	1/2	−1/2	
e^R	0	0	$\Leftarrow$ WEAK ISOSPIN SINGLET

REQUIRE THEORY TO BE INVARIANT UNDER

$$\boxed{\Psi_e^L \xrightarrow{SU(2)} \exp\{i g \Theta_i(x) t_i\} \Psi_e^L} \equiv U \Psi_e^L$$

$U \in SU(2)$

$$[t_i, t_j] = i \varepsilon_{ijk} t_k \qquad t_i = \frac{\tau_i}{2}$$

$\Downarrow$

INVARIANCE OF $\mathcal{L}_0$ REQUIRES INTRODUCTION OF 3 VECTOR (GAUGE) FIELDS W_i^μ ($i = 1,2,3$)

$$\boxed{\partial^\mu \Psi_e^L \Longrightarrow \underbrace{(\partial^\mu + i g t_i W_i^\mu)}_{\equiv D^\mu} \Psi_e^L}$$

SUCH THAT $\qquad \mathbb{W}^\mu \equiv t_i W_i^\mu$

$$D^\mu \Psi_e^L \xrightarrow{SU(2)} \exp\{i g \Theta_i t_i\} D^\mu \Psi_e^L \equiv U (D^\mu \Psi_e^L)$$

$$\bar{\Psi}_e^L \gamma_\mu (D^\mu \Psi_e^L) \xrightarrow{SU(2)} \bar{\Psi}_e \gamma_\mu (D^\mu \Psi_e)$$

LEADS TO TF. PROPERTY OF GAUGE FIELDS

$$U(D^\mu \Psi_e^L) = \partial^\mu (U \Psi_e^L) + i g \mathbb{W}'^\mu (U \Psi_e^L)$$

$$\parallel$$

$$U \left[\partial^\mu \Psi_e^L + i g \mathbb{W}^\mu \Psi_e^L \right]$$

$$U\left[\partial^\mu \Psi_e^L + ig\,\mathbb{W}^\mu \Psi_e^L\right]$$

$$= (\partial^\mu U)\,\Psi_e^L + U(\partial^\mu \Psi_e^L) + ig\,\mathbb{W}'^\mu (U\Psi_e^L)$$

$$\Downarrow$$

$$\mathbb{W}'^\mu U = U\,\mathbb{W}^\mu + \frac{i}{g}(\partial^\mu U)$$

$$\boxed{\mathbb{W}^\mu \xrightarrow[SU(2)]{} \mathbb{W}'^\mu = U\,\mathbb{W}^\mu U^{-1} + \frac{i}{g}(\partial^\mu U)\,U^{-1}}$$

$\downarrow$ INFINITESIMAL

$$U \simeq 1 + ig\,\theta_i\, t_i$$

$$t_i W_i'^\mu = t_i W_i^\mu + ig\,\underbrace{[t_j, t_k]}_{i\varepsilon_{ijk} t_i}\,\theta_j W_k^\mu$$

$$+ \frac{i}{g}\, ig\,(\partial^\mu \theta_i)\, t_i$$

$$\boxed{W_i'^\mu = W_i^\mu - (\partial^\mu \theta_i) - g\,\varepsilon_{ijk}\,\theta_j W_k^\mu}$$

AS IN QED

NEW TERM DUE TO NON-ABELIAN GAUGE GROUP SU(2)

$$\hookrightarrow \quad \mathcal{L}_0 + \mathcal{L}_{INT}$$

$$= \bar{\Psi}_\ell^L \, i \, \gamma^\mu D_\mu \Psi_\ell^L + \bar{\nu}_\ell^R \, i \, \gamma^\mu \partial_\mu \nu_\ell^R$$

$$= \mathcal{L}_0 - g \, \bar{\Psi}_\ell^L \gamma^\mu t_i \Psi_\ell^L \, W_{i\mu}$$

$$\mathcal{L}_{INT} = - g \Big\{ \bar{\Psi}_\ell^L \gamma^\mu t_1 \Psi_\ell^L \, W_{1\mu} + \bar{\Psi}_\ell^L \gamma^\mu t_2 \Psi_\ell^L \, W_{2\mu}$$
$$+ \bar{\Psi}_\ell^L \gamma^\mu t_3 \Psi_\ell^L \, W_{3\mu} \Big\}$$

INTRODUCE CHARGED GAUGE FIELDS

$$W^\mu \equiv \frac{1}{\sqrt{2}} \left(W_1^\mu - i W_2^\mu \right)$$ ANNIHILATES W^+, CREATES W^-

$$W^{\mu +} \equiv \frac{1}{\sqrt{2}} \left(W_1^\mu + i W_2^\mu \right)$$ ANNIHILATES W^-, CREATES W^+

$$\downarrow$$

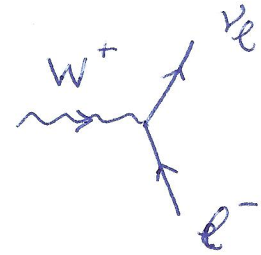
$$W_1^\mu = \frac{1}{\sqrt{2}} \left(W^\mu + W^{\mu +} \right)$$

$$W_2^\mu = \frac{i}{\sqrt{2}} \left(W^\mu - W^{\mu +} \right)$$

$$\mathcal{L}_{INT} = - g \Big\{ \bar{\Psi}_\ell^L \gamma^\mu \left(t_1 + i t_2 \right) \Psi_\ell^L \, W_\mu \frac{1}{\sqrt{2}}$$
$$+ \bar{\Psi}_\ell^L \gamma^\mu \left(t_1 - i t_2 \right) \Psi_\ell^L \, W_\mu^+ \frac{1}{\sqrt{2}}$$
$$+ \bar{\Psi}_\ell^L \gamma^\mu t_3 \Psi_\ell^L \, W_{3\mu} \Big\}$$

$$t_{\pm} = t_1 \pm i t_2$$

$$\bar{\Psi}_{\ell}^{L} \gamma^{\mu} t_{+} \Psi_{\ell}^{L} = \bar{\psi}_{\nu_e}^{L} \gamma^{\mu} \psi_{e}^{L}$$



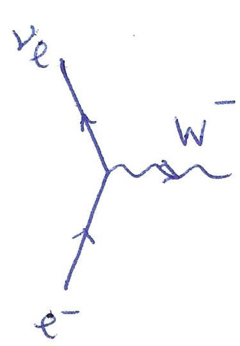
$$\bar{\Psi}_{\ell}^{L} \gamma^{\mu} t_{-} \Psi_{\ell}^{L} = \bar{\psi}_{e}^{L} \gamma^{\mu} \psi_{\nu_e}^{L}$$

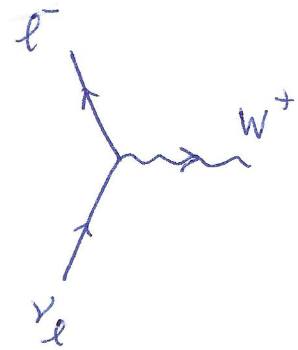
$$= \bar{\psi}_{e} \tfrac{1}{2}(1+\gamma_5) \gamma^{\mu} \tfrac{1}{2}(1-\gamma_5) \psi_{\nu_e}$$

$$= \frac{1}{2} \bar{\psi}_{e} \gamma^{\mu}(1-\gamma_5) \psi_{\nu_e}$$

$$\therefore \quad \boxed{\begin{aligned} \mathcal{L}_{INT}^{SU(2)} = & -\frac{g}{2\sqrt{2}} \bar{\psi}_{\nu_e} \gamma^{\mu}(1-\gamma_5) \psi_e W_{\mu} \\ & -\frac{g}{2\sqrt{2}} \bar{\psi}_{e} \gamma^{\mu}(1-\gamma_5) \psi_{\nu_e} W_{\mu}^{+} \\ & -\frac{g}{2(2)} \left\{ \bar{\psi}_{\nu_e} \gamma^{\mu}(1-\gamma_5) \psi_{\nu_e} - \bar{\psi}_{e} \gamma^{\mu}(1-\gamma_5) \psi_{e} \right\} W_{3\mu} \end{aligned}}$$

FIRST 2 TERMS DESCRIBE CHARGED WEAK INTERACTION





FEYNMAN RULE

$$\boxed{-i \frac{g}{2\sqrt{2}} \gamma^{\mu}(1-\gamma_5)}$$

• WEAK HYPERCHARGE : $U(1)_Y$

2 NEUTRAL CURRENTS

→ $J_3^\mu = \bar{\Psi}_\ell^L \gamma^\mu t_3 \Psi_\ell^L$ ⇝ COUPLES TO W_3

$$= \frac{1}{2} \bar{\psi}_{\nu_\ell}^L \gamma^\mu \psi_{\nu_\ell}^L - \frac{1}{2} \bar{\psi}_\ell^L \gamma^\mu \psi_\ell^L$$

→ $J_{em}^\mu = \bar{\psi}_\ell^L \gamma^\mu Q \psi_\ell^L + \bar{\psi}_\ell^R \gamma^\mu Q \psi_\ell^R$ ⇝ COUPLES TO γ

INTRODUCE 'WEAK HYPERCHARGE' Y SUCH THAT

$$\underset{\substack{\uparrow \\ \text{ELECTRIC} \\ \text{CHARGE}}}{Q} = \underset{\substack{\uparrow \\ \text{WEAK} \\ \text{ISOSPIN}}}{t_3} + \frac{1}{2} \underset{\substack{\uparrow \\ \text{WEAK} \\ \text{HYPERCHARGE}}}{Y}$$

	t	t_3	Y	Q
ν_ℓ^L	1/2	+1/2	−1	0
ℓ^L	1/2	−1/2	−1	−1
ℓ^R	0	0	−2	−1

$$\frac{1}{2} J_Y^\mu = J_{em}^\mu - J_3^\mu$$

↳ INVARIANCE OF $\mathcal{L}$ UNDER $U(1)_Y$

$$\boxed{\psi_e \xrightarrow{U(1)_Y} \exp\left\{ i g' \chi(x) \frac{Y}{2} \right\} \psi_e}$$

$$\Downarrow$$

$$\boxed{\partial^\mu \psi_e \Rightarrow \left(\partial^\mu + i g' \frac{Y}{2} B^\mu \right) \psi_e}$$

GAUGE FIELD

$$B^\mu \xrightarrow{U(1)} B^\mu - \partial^\mu \chi$$

$$\boxed{\begin{aligned} \mathcal{L}^{U(1)_Y}_{INT} = - g' \Big\{ & \bar{\psi}^L_{\nu_e} \gamma^\mu \left(\frac{-1}{2}\right) \psi^L_{\nu_e} + \bar{\psi}^L_e \gamma^\mu \left(-\frac{1}{2}\right) \psi^L_e \\ & + \bar{\psi}^R_e \gamma^\mu \left(-\frac{2}{2}\right) \psi^R_e \Big\} B_\mu \end{aligned}}$$

NEUTRAL CURRENT

$$\begin{aligned} \hookrightarrow \mathcal{L}^{NC}_{INT} = & - g \bar{\Psi}^L_e \gamma^\mu t_3 \Psi^L_e \, W_{3\mu} \\ & - g' \bar{\Psi}^L_e \gamma^\mu \left(\frac{Y}{2}\right) \Psi^L_e B_\mu - g' \bar{\psi}^R_e \gamma^\mu \left(\frac{Y}{2}\right) \psi^R_e B_\mu \end{aligned}$$

$$\downarrow$$

REPLACE $\frac{1}{2} Y = Q - t_3$

$$\mathcal{L}_{INT}^{NC} = -\bar{\Psi}_e^L \gamma^\mu t_3 \Psi_e^L \left(g W_{3\mu} - g' B_\mu \right)$$

$$- g' B_\mu \underbrace{\left(\bar{\psi}_e^L \gamma^\mu Q \psi_e^L + \bar{\psi}_e^R \gamma^\mu Q \psi_e^R \right)}_{\bar{\psi}_e \gamma^\mu Q \psi_e \, \equiv \, J^\mu_{em}}$$

E.M. CURRENT

$$\boxed{\begin{aligned} \mathcal{L}_{INT}^{NC} = &- \bar{\Psi}_e^L \gamma^\mu t_3 \Psi_e^L \left(g W_{3\mu} - g' B_\mu \right) \\ &- J^\mu_{em} \, g' B_\mu \end{aligned}}$$

PHYSICAL GAUGE BOSON FIELDS (γ, Z^0)
ARE LINEAR COMBINATIONS OF (W_3, B)
SUCH THAT $g W_{3\mu} - g' B_\mu$ DOES NOT CONTAIN γ
(BECAUSE IT COUPLES TO NEUTRINO)

ELECTROWEAK UNIFICATION

- **ELECTROWEAK UNIFICATION** : GLASHOW - WEINBERG - SALAM MODEL (1967, 1968)

↳ PHYSICAL GAUGE BOSON FIELDS ARE MIXTURE OF W_3, B

⇓

MIXING ANGLE : **WEINBERG ANGLE Θ_W**

→ PHYSICAL GAUGE BOSON FIELDS

$$\boxed{\begin{pmatrix} W_3^\mu \\ B^\mu \end{pmatrix} = \begin{pmatrix} \cos\Theta_W & \sin\Theta_W \\ -\sin\Theta_W & \cos\Theta_W \end{pmatrix} \begin{pmatrix} Z^\mu \\ A^\mu \end{pmatrix}}$$

↳ $g\, W_{3\mu} - g'\, B_\mu$ SHOULD NOT CONTAIN A_μ

$$= \left(g\cos\Theta_W + g'\sin\Theta_W\right) Z_\mu + \underbrace{\left(g\sin\Theta_W - g'\cos\Theta_W\right)}_{=\,0} A_\mu$$

$$\boxed{g\sin\Theta_W = g'\cos\Theta_W} \qquad \tan\Theta_W = \frac{g'}{g}$$

↳ $g'\, B_\mu$

$$= -\,g'\sin\Theta_W\, Z_\mu + g'\cos\Theta_W\, A_\mu$$

$g'\, B_\mu$ COUPLES TO E.M. CURRENT $\Longrightarrow$ $\boxed{g'\cos\Theta_W = e}$

↳ $\mathcal{L}_{INT}^{NC}$ IN TERMS OF A^μ & Z^μ

$$\mathcal{L}_{INT}^{NC} = - \bar{\Psi}_\ell^L \gamma^\mu t_3 \Psi_\ell^L \left(\frac{g}{\cos\theta_W} \right) Z_\mu$$

$$- J_{em}^\mu \left(- \frac{g}{\cos\theta_W} \sin^2\theta_W \right) Z_\mu - e J_{em}^\mu A_\mu$$

⇓

$$\boxed{\mathcal{L}_{INT}^{NC} = - e J_{em}^\mu A_\mu - \frac{g}{\cos\theta_W} \underbrace{\left(J_3^\mu - \sin^2\theta_W J_{em}^\mu \right)}_{J_{NC}^\mu} Z_\mu}$$

⇑

NEUTRAL WEAK CURRENT

1° TERM : E.M. INTERACTION

2° TERM : NEUTRAL WEAK INTERACTION

↓

NEUTRAL WEAK CURRENT J_{NC}^μ

$$J_{NC}^\mu = \frac{1}{4} \bar{\psi}_{\nu_e} \gamma^\mu (1 - \gamma_5) \psi_{\nu_e}$$

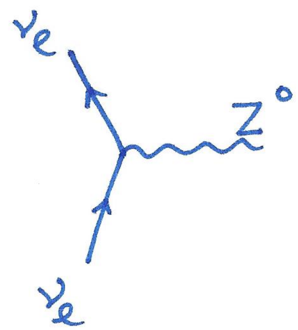
$$- \frac{1}{4} \bar{\psi}_e \gamma^\mu (1 - \gamma_5) \psi_e + \sin^2\theta_W \bar{\psi}_e \gamma^\mu \psi_e .$$

↓

(LEPTON HAS NEGATIVE CHARGE)

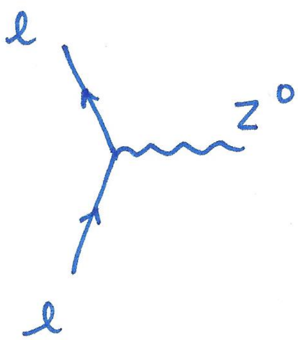
$$\boxed{J^{\mu}_{NC} = \frac{1}{4}\,\bar{\psi}_{\nu_e}\,\gamma^{\mu}(1-\gamma_5)\,\psi_{\nu_e} - \frac{1}{4}\,\bar{\psi}_{e}\,\gamma^{\mu}\left(\left[1-4\sin^2\theta_W\right]-\gamma_5\right)\psi_{e}}$$

↳ FEYNMAN RULES : NEUTRAL WEAK INTERACTION



$$\boxed{-\,i\,\frac{g}{4\cos\theta_W}\,\gamma^{\mu}(1-\gamma_5)}$$

(V – A)



$$\boxed{+\,i\,\frac{g}{4\cos\theta_W}\,\gamma^{\mu}\left(\underbrace{\left[1-4\sin^2\theta_W\right]}_{c_V}-\gamma_5\right)}$$

ELECTROWEAK INTERACTIONS

DESCRIBED BY 1 COUPLING : e

& 1 MIXING ANGLE $\sin\theta_W$ ⇒ TO BE DETERMINED BY EXPERIMENT

g & g' FOLLOW FROM $\boxed{g\sin\theta_W = g'\cos\theta_W = e}$

- $\boxed{SU(2) \times U(1)_Y \quad \text{GAUGE BOSON LAGRANGIAN}}$

↳ $$\mathcal{L}_{SU(2)} = -\frac{1}{4} F_i^{\mu\nu} F_{i\mu\nu}$$

$$F_i^{\mu\nu} = \partial^\mu W_i^\nu - \partial^\nu W_i^\mu - g\,\varepsilon_{ijk} W_j^\mu W_k^\nu$$

↳ $$\mathcal{L}_{U(1)_Y} = -\frac{1}{4} B^{\mu\nu} B_{\mu\nu}$$

↳ CHARGED $W^\pm$ FIELDS

$$W^{\pm\mu} \equiv \frac{1}{\sqrt{2}} \left(W_1^\mu \mp i W_2^\mu \right)$$

$\hat{W}^\pm$ ANNIHILATES $W^\pm$

$$W_1^\mu = \frac{1}{\sqrt{2}} \left(W^{+\mu} + W^{-\mu} \right)$$

$$W_2^\mu = \frac{i}{\sqrt{2}} \left(W^{+\mu} - W^{-\mu} \right)$$

$$\begin{aligned}\mathcal{L}_{SU(2)} = &-\frac{1}{2}(\partial^\mu W_1^\nu)(\partial_\mu W_{1\nu}) + \frac{1}{2}(\partial^\mu W_1^\nu)(\partial_\nu W_{1\mu}) \\ &-\frac{1}{2}(\partial^\mu W_2^\nu)(\partial_\mu W_{2\nu}) + \frac{1}{2}(\partial^\mu W_2^\nu)(\partial_\nu W_{2\mu}) \\ &-\frac{1}{2}(\partial^\mu W_3^\nu)(\partial_\mu W_{3\nu}) + \frac{1}{2}(\partial^\mu W_3^\nu)(\partial_\nu W_{3\mu}) \\ &+ \mathcal{L}_{3W} + \mathcal{L}_{4W}\end{aligned}$$

$$\hookrightarrow \mathcal{L}_{SO(2)} + \mathcal{L}_{U(1)_Y}$$

$$= -\frac{1}{2}\left(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+\right)\left(\partial^\mu W^{-\nu} - \partial^\nu W^{-\mu}\right)$$

$$-\frac{1}{4} W_3^{\mu\nu} W_{3\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu}$$

$$+ \mathcal{L}_{3W} + \mathcal{L}_{4W}$$

WITH $W_3^{\mu\nu} \equiv \partial^\mu W_3^\nu - \partial^\nu W_3^\mu$

$B^{\mu\nu} \equiv \partial^\mu B^\nu - \partial^\nu B^\mu$

ELECTROWEAK UNIFICATION

$$\begin{pmatrix} W_3^\mu \\ B^\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} Z^\mu \\ A^\mu \end{pmatrix}$$

$$W_3^{\mu\nu} W_{3\mu\nu} + B^{\mu\nu} B_{\mu\nu} = Z^{\mu\nu} Z_{\mu\nu} + F^{\mu\nu} F_{\mu\nu}$$

WITH $Z^{\mu\nu} \equiv \partial^\mu Z^\nu - \partial^\nu Z^\mu$

$F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu$

$$\boxed{\begin{aligned}&\mathcal{L}_{SU(2)} + \mathcal{L}_{U(1)_Y}\\ &= -\frac{1}{2}\left(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+\right)\left(\partial^\mu W^{-\nu} - \partial^\nu W^{-\mu}\right)\\ &\quad -\frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \mathcal{L}_{3W} + \mathcal{L}_{4W}\end{aligned}}$$

$$\begin{aligned}\mathcal{L}_{3W} &= \frac{g}{2} \varepsilon_{ijk} \left(\partial^\mu W_i^\nu - \partial^\nu W_i^\mu\right) W_{j\mu} W_{k\nu}\\
&= g \Big\{ \left(\partial^\mu W_1^\nu - \partial^\nu W_1^\mu\right) W_{2\mu} W_{3\nu}\\
&\qquad - \left(\partial^\mu W_2^\nu - \partial^\nu W_2^\mu\right) W_{1\mu} W_{3\nu}\\
&\qquad + \left(\partial^\mu W_3^\nu - \partial^\nu W_3^\mu\right) W_{1\mu} W_{2\nu} \Big\}\\
&= ig \Big\{ -\left(\partial^\mu W^{+\nu} - \partial^\nu W^{+\mu}\right) W_\mu^- W_{3\nu}\\
&\qquad + \left(\partial^\mu W^{-\nu} - \partial^\nu W^{-\mu}\right) W_\mu^+ W_{3\nu}\\
&\qquad - \left(\partial^\mu W_3^\nu - \partial^\nu W_3^\mu\right) W_\mu^+ W_\nu^- \Big\}\end{aligned}$$

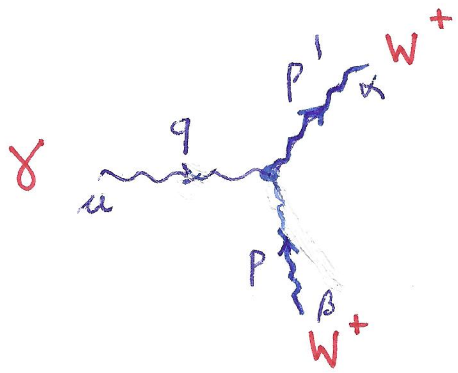
$$\downarrow \quad W_3^\mu = \cos\theta_W Z^\mu + \sin\theta_W A^\mu$$

$$\mathcal{L}_{3W} = +ig\cos\theta_W \Big[-(\partial^\mu W^{+\nu} - \partial^\nu W^{+\mu}) W^-_\mu Z_\nu$$
$$+ (\partial^\mu W^{-\nu} - \partial^\nu W^{-\mu}) W^+_\mu Z_\nu$$
$$- Z^{\mu\nu} W^+_\mu W^-_\nu \Big]$$
$$+ ig\sin\theta_W \Big[-(\partial^\mu W^{+\nu} - \partial^\nu W^{+\mu}) W^-_\mu A_\nu$$
$$+ (\partial^\mu W^{-\nu} - \partial^\nu W^{-\mu}) W^+_\mu A_\nu$$
$$- F^{\mu\nu} W^+_\mu W^-_\nu \Big]$$

$$\boxed{\begin{aligned} \mathcal{L}_{3W} = +ie\frac{\cos\theta_W}{\sin\theta_W} \Big[& -Z^{\mu\nu} W^+_\mu W^-_\nu \\ & + (\partial^\mu W^{-\nu} - \partial^\nu W^{-\mu}) W^+_\mu Z_\nu \\ & - (\partial^\mu W^{+\nu} - \partial^\nu W^{+\mu}) W^-_\mu Z_\nu \Big] \\ - ie \Big[& F^{\mu\nu} W^+_\mu W^-_\nu \\ & + (\partial^\mu W^{+\nu} - \partial^\nu W^{+\mu}) W^-_\mu A_\nu \\ & - (\partial^\mu W^{-\nu} - \partial^\nu W^{-\mu}) W^+_\mu A_\nu \Big] \end{aligned}}$$

↳ FEYNMAN RULES : 3 GAUGE BOSON COUPLINGS

S-MATRIX ELEMENT



$$\boxed{ie\left\{(p+p')^{\mu} g^{\alpha\beta} - g^{\alpha\mu} p'^{\beta} - g^{\beta\mu} p^{\alpha} + \left(q^{\alpha} g^{\beta\mu} - q^{\beta} g^{\alpha\mu}\right)\right\}}$$

$q \equiv p' - p$

<u>NOTE</u>

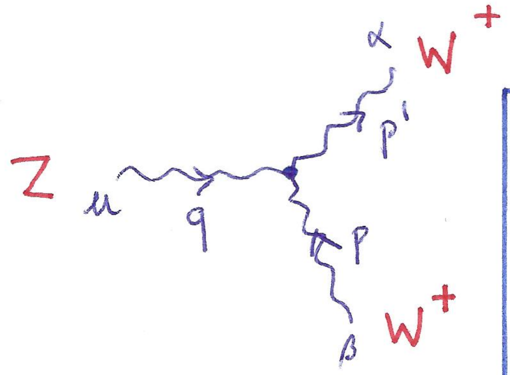
- NAIVE MINIMAL SUBSTITUTION IN CHARGED SPIN 1 (PROCA) LAGRANGIAN WOULD ONLY YIELD
$$ie\left\{(p+p')^{\mu} g^{\alpha\beta} - g^{\alpha\mu} p'^{\beta} - g^{\beta\mu} p^{\alpha}\right\}$$
- TERM $-ie\, F^{\mu\nu} W^{+}_{\mu} W^{-}_{\nu}$ IN $\mathcal{L}_{3W}$ YIELDS <u>NON</u>-MINIMAL TERM IN VERTEX
$$ie\left(q^{\alpha} g^{\beta\mu} - q^{\beta} g^{\alpha\mu}\right)$$

THIS TERM IS RESPONSIBLE FOR <u>$g = 2$</u> (g-FACTOR)

↳ MAGN. MOMENT $\Rightarrow \mu_W = g \cdot \left(\frac{e}{2M_W}\right)$

↳ QUADRUPOLE MOMENT $\Rightarrow Q_W = -\frac{e}{M_W^2}$

(MINIMAL TERM WOULD GIVE $g = 1$, $Q_W = 0$)



$$\boxed{+ie\frac{\cos\theta_W}{\sin\theta_W}\left\{(p+p')^\mu g^{\alpha\beta} - g^{\alpha\mu}p'^\beta - g^{\beta\mu}p^\alpha + \left(q^\alpha g^{\beta\mu} - q^\beta g^{\alpha\mu}\right)\right\}}$$

$$\hookrightarrow \quad \mathcal{L}_{4W} = -\frac{g^2}{4}\,\varepsilon_{ijk}\,\varepsilon_{ilm}\; W_j^\mu W_k^\nu W_{l\mu} W_{m\nu}$$

$$\downarrow \qquad \varepsilon_{ijk}\,\varepsilon_{ilm} = \delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl}$$

$$= -\frac{g^2}{4}\left\{\left(W_j^\mu W_{j\mu}\right)\left(W_k^\nu W_{k\nu}\right) - \left(W_j^\mu W_{j\nu}\right)\left(W_{k\mu} W_k^\nu\right)\right\}$$

$$= -\frac{g^2}{4}\left\{\left(2W^{+\mu}W^-_\mu + W_3^\mu W_{3\mu}\right)\cdot\left(2W^{+\nu}W^-_\nu + W_3^\nu W_{3\nu}\right) - \left(2W^{+\mu}W^-_\nu + W_3^\mu W_{3\nu}\right)\cdot\left(W^+_\mu W^{-\nu} + W^{+\nu}W^-_\mu + W_{3\mu}W_3^\nu\right)\right\}$$

$$\mathcal{L}_{4W} = \frac{g^2}{2} \Big\{ W^{+\mu} W^{+}_{\mu} W^{-\nu} W^{-}_{\nu} - W^{+\mu} W^{-}_{\mu} W^{+\nu} W^{-}_{\nu}$$

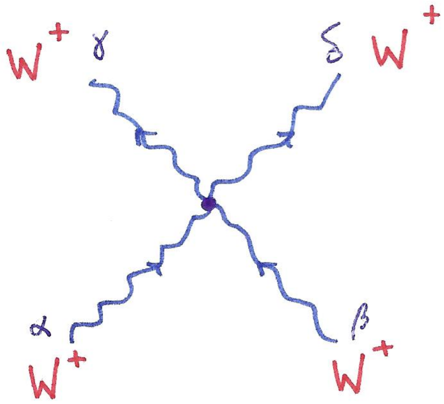
$$- 2 \left(W^{+\mu} W^{-}_{\mu} \right) \left(W_3 \right)^2$$

$$+ 2 \left(W^{+\mu} W_{3\mu} \right) \left(W^{-\nu} W_{3\nu} \right) \Big\}$$

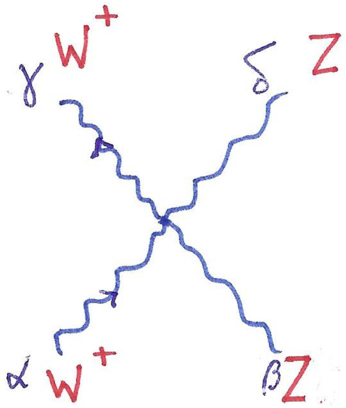
$$\Big\downarrow \quad W_3^{\mu} = \cos\theta_W \, Z^{\mu} + \sin\theta_W \, A^{\mu}$$

$$\boxed{\begin{aligned} \mathcal{L}_{4W} = {} & \frac{e^2}{2\sin^2\theta_W} \left(W^{+\mu} W^{+}_{\mu} W^{-\nu} W^{-}_{\nu} - W^{+\mu} W^{-}_{\mu} W^{+\nu} W^{-}_{\nu} \right) \\ & + e^2 \left(A_{\mu} W^{+\mu} A_{\nu} W^{-\nu} - A_{\mu}^2 W^{+\nu} W^{-}_{\nu} \right) \\ & + e^2 \frac{\cos\theta_W}{\sin\theta_W} \Big(W^{+\mu} A_{\mu} W^{-\nu} Z_{\nu} + W^{+\mu} Z_{\mu} W^{-\nu} A_{\mu} \\ & \qquad - 2 W^{+\mu} W^{-}_{\mu} Z^{\nu} A_{\nu} \Big) \\ & + e^2 \frac{\cos^2\theta_W}{\sin^2\theta_W} \left(Z_{\mu} W^{+\mu} Z_{\nu} W^{-\nu} - Z_{\mu}^2 W^{+\nu} W^{-}_{\nu} \right) \end{aligned}}$$

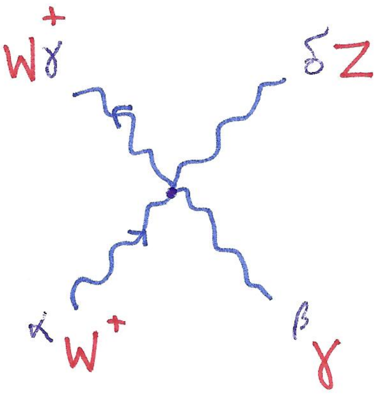
↳ FEYNMAN RULES: 4 GAUGE BOSON COUPLINGS



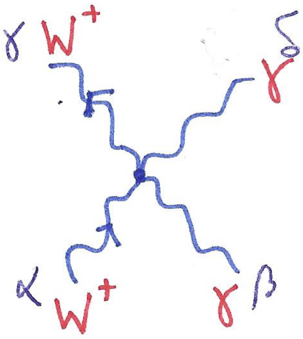
$$\frac{ie^2}{\sin^2\theta_W}\left(2g^{\alpha\beta}g^{\gamma\delta}-g^{\alpha\gamma}g^{\beta\delta}-g^{\alpha\delta}g^{\beta\gamma}\right)$$



$$i\,\frac{e^2\cos^2\theta_W}{\sin^2\theta_W}\left(g^{\alpha\beta}g^{\gamma\delta}+g^{\alpha\delta}g^{\beta\gamma}-2g^{\beta\delta}g^{\alpha\gamma}\right)$$



$$ie^2\,\frac{\cos\theta_W}{\sin\theta_W}\left(g^{\alpha\beta}g^{\gamma\delta}+g^{\alpha\delta}g^{\beta\gamma}-2g^{\beta\delta}g^{\alpha\gamma}\right)$$



$$ie^2\left(g^{\alpha\beta}g^{\gamma\delta}+g^{\alpha\delta}g^{\beta\gamma}-2g^{\beta\delta}g^{\alpha\gamma}\right)$$