NON-ABELIAN GAUGE THEORY
QCD

⇒ ABELIAN GAUGE THEORY, QED

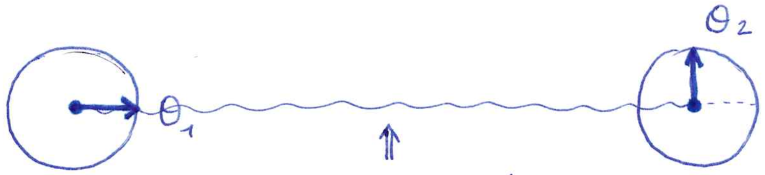
- FERMION (SPIN 1/2)

DIRAC EQ. $(i\gamma^\mu \partial_\mu - m)\psi(x) = 0$

$$\mathcal{L}_0 = \bar{\psi}(x)\,(i\gamma^\mu\partial_\mu - m)\,\psi(x)$$

WITH $\partial_\mu = \frac{\partial}{\partial x^\mu} = \left(\frac{\partial}{\partial t}, \vec{\nabla}\right)$

- LOCAL PHASE TRANSFORMATION



↑ SIGNAL WHICH COMMUNICATES PHASE DIFFERENCE ("PHOTON")

$\Delta\theta = \theta_2 - \theta_1$

PHASE DIFFERENCE

$$\psi(x) \longrightarrow e^{i\theta(x)}\,\psi(x)$$

$$\partial_\mu\psi(x) \longrightarrow e^{i\theta(x)}\left[\partial_\mu\psi + i(\partial_\mu\theta)\psi\right]$$

$$\bar{\psi}\psi \longrightarrow \bar{\psi}\psi$$

$$\bar{\psi}\gamma^\mu(\partial_\mu\psi) \longrightarrow \bar{\psi}\gamma^\mu(\partial_\mu\psi) + i(\partial_\mu\theta)\,\bar{\psi}\gamma^\mu\psi$$

$$\mathcal{L}_0 \longrightarrow \mathcal{L}_0 - (\partial_\mu \theta) \underbrace{\bar{\psi} \gamma^\mu \psi}_{\text{VECTOR}}$$

- <u>INVARIANCE UNDER LOCAL PHASE TRANSFORMATION</u>

INTRODUCE VECTOR FIELD (GAUGE FIELD)

$$\partial_\mu \Rightarrow D_\mu \equiv \partial_\mu + i e A_\mu$$

$\Uparrow$ COVARIANT DERIVATIVE

e: COUPLING STRENGTH OF FERMION TO GAUGE FIELD (ELECTRIC CHARGE)

$$A_\mu \longrightarrow A_\mu - \frac{1}{e} \partial_\mu \theta$$

$$D_\mu \psi(x) \rightarrow e^{i\theta(x)} \left[\partial_\mu \psi + \underbrace{i(\partial_\mu \theta)\psi + ie A'_\mu \psi}_{\Downarrow \; A'_\mu = A_\mu - \frac{1}{e}\partial_\mu \theta} \right]$$

$$= e^{i\theta(x)} \left[\partial_\mu \psi + ie A_\mu \psi \right]$$

$$= e^{i\theta(x)} D_\mu \psi(x)$$

$$\bar{\psi}(x) \gamma^\mu D_\mu \psi(x) \longrightarrow \bar{\psi}(x) \gamma^\mu D_\mu \psi$$ INVARIANT !

TO GET THEORY WHICH HAS SYMMETRY UNDER LOCAL PHASE TRANSFORMATION

REPLACE $\partial_\mu \psi \Rightarrow D_\mu \psi$

$\Downarrow$

REPLACE $\mathcal{L}_0 \Rightarrow \bar{\psi}(x)\,(i\gamma^\mu D_\mu - m)\,\psi(x)$

$$= \underbrace{\bar{\psi}(x)\,(i\gamma^\mu \partial_\mu - m)\,\psi(x)}_{\mathcal{L}_0} \;+\; \underbrace{- e\,\bar{\psi}\gamma^\mu\psi A_\mu}_{\mathcal{L}_{INT}}$$

$\mathcal{L}_0 + \mathcal{L}_{INT}$ IS INVARIANT UNDER LOCAL PHASE TF.

U(1) GAUGE TF

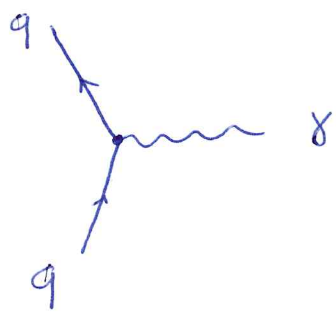
$\mathcal{L}_0 + \mathcal{L}_{INT}$ HAS U(1) GAUGE SYMMETRY

- COUPLING TO PHOTON (GAUGE FIELD)

↳ DESCRIBED BY $\mathcal{L}_{INT} = - e\,\bar{\psi}\gamma^\mu\psi A_\mu$

↳ VECTOR POTENTIAL

$\psi = q$



FEYNMAN RULE

$$\boxed{-ie\,\gamma^\mu.}$$

- GEOMETRIC INTERPRETATION OF COVARIANT DERIVATIVE

→ INFINITESIMAL DISTANCE dx^μ BETWEEN 2 POINTS

$$dx^\mu \cdot \partial_\mu \psi(x) = \psi(x+dx) - \psi(x)$$

$$\rightarrow \quad dx^\mu \cdot D_\mu \psi(x) = dx^\mu \left(\partial_\mu \psi(x) + ie\, A_\mu(x)\, \psi(x) \right)$$

↓ TAKE AS GAUGE FIELD

$$A_\mu = -\frac{1}{e}\partial_\mu \theta$$

$$= dx^\mu\, \partial_\mu \psi(x) - i\, dx^\mu (\partial_\mu \theta)\, \psi(x)$$

$$= \psi(x+dx) - \psi(x) - i\left[\theta(x+dx) - \theta(x)\right]\psi(x)$$

$$= \psi(x+dx) - \left[1 + i\left(\theta(x+dx) - \theta(x)\right)\right]\psi(x)$$

$$\simeq \psi(x+dx) - \underbrace{\exp\left\{ i\theta(x+dx) - i\,\theta(x)\right\}}_{\Phi(x+dx,\, x)}\psi(x)$$

FACTOR WHICH COMPENSATES PHASE DIFFERENCE IN FIELD $\psi(x)$ AT POINTS x AND $x+dx$

$\Phi(x+dx, x)$

$$\boxed{\begin{aligned}\Phi(x+dx,\,x) &\equiv \exp\{i\,dx^\mu\,\partial_\mu\theta\}\\ &= \exp\{-ie\,dx^\mu A_\mu(x)\}\end{aligned}}$$

"PARALLEL TRANSPORT"

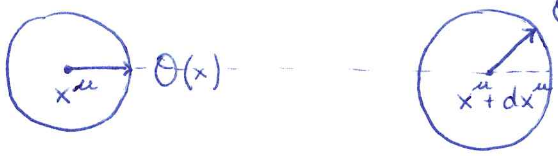
COMPARE FIELDS COMPENSATED FOR THEIR PHASE DIFFERENCE AT 2 POINTS

INSTEAD OF FIELDS THEMSELVES

$$\boxed{dx^\mu D_\mu \psi(x) = \psi(x+dx) - \Phi(x+dx,\,x)\,\psi(x)}$$

$$-e\,A_\mu(x) = \partial_\mu\theta$$

↳ GAUGE FIELD → INTERPRETATION AS RELATIVE ORIENTATION (IN DIRECTION μ) OF PHASE AT 2 NEIGHBOURING POINTS



$\theta(x+dx) = \theta(x) + dx^\mu(\partial_\mu\theta)$

→ FOR A FINITE PATH P :

$$\boxed{\Phi_P(y,\,x) \equiv \exp\left\{-ie\int_P dx^\mu A_\mu(x)\right\}}$$

$\bar\psi(y)\,\Phi_P(y,x)\,\psi(x)$ IS GAUGE INVARIANT $\big(U(1)\big)$

CAN ONE SHOW EXPERIMENTALLY THAT A GAUGE FIELD (GAUGE
IS EQUIVALENT TO A PHASE DIFFERENCE FOR FIELD ψ ?

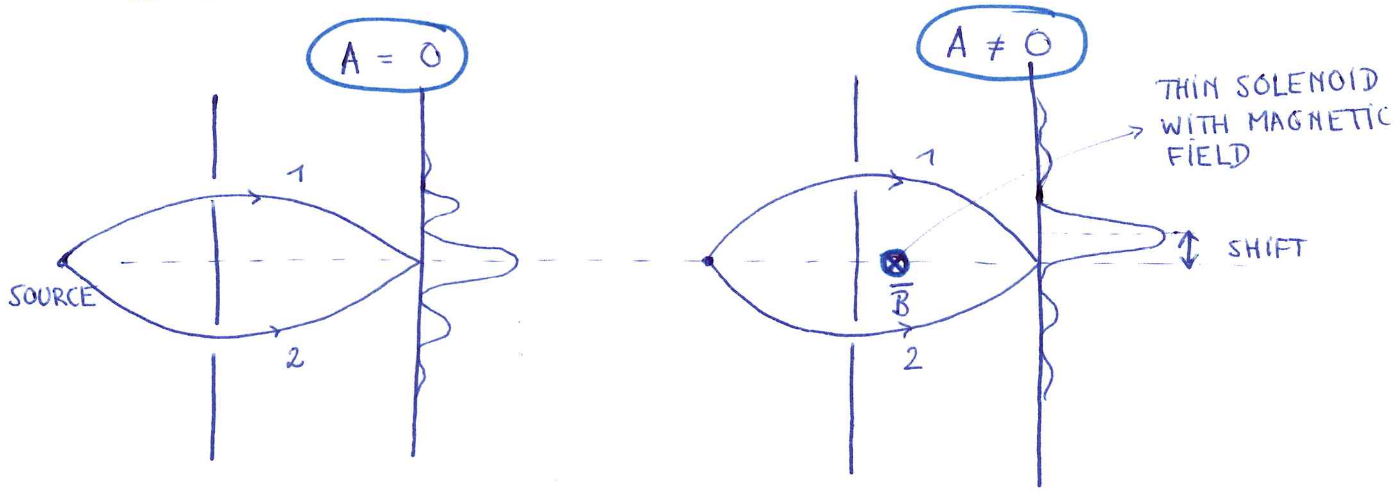
⇕

IS EFFECT OF GAUGE FIELD OBSERVABLE

YES !

AHARONOV - BOHM EXPERIMENT

2 SLIT EXPERIMENT



NO PHASE DIFFERENCE FOR e^- ALONG 1 OR 2

PHASE DIFFERENCE FOR e^- ALONG 1 OR 2

$$\Delta\theta = \left(-e\int_{P2} dx^\mu A_\mu\right) - \left(-e\int_{P1} dx^\mu A_\mu\right)$$

$$= -e\oint dx^\mu A_\mu$$

C → CLOSED CONTOUR (CCW)

STOKES LAW ↓

$$= -e\,\Phi_M$$

↳ MAGNETIC FLUX THROUGH LOOP

⇓

INTERFERENCE PATTERN CHANGED !

IN PRESENCE OF GAUGE FIELD A,

BY GOING AROUND A CLOSED CURVE,

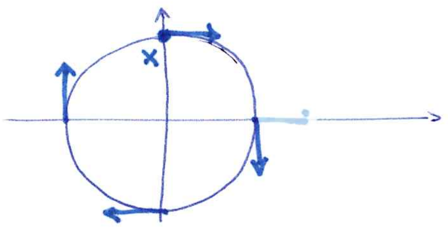
THE PHASE OF FERMION FIELD HAS CHANGED

BY AMOUNT $-e\Phi_M$

GEOMETRIC ANALOGY

→ PERSON LIVING IN FLAT 2 DIM WORLD

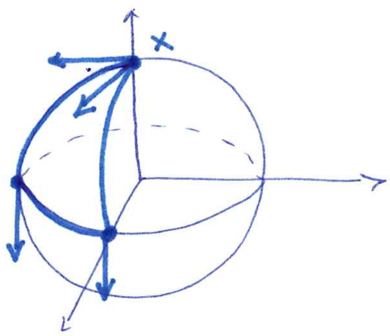
GOING AROUND CLOSED CURVE, HOLDING AN ARROW



$\Delta\theta = 0$

→ PERSON LIVING IN CURVED 2 DIM WORLD (SURFACE OF SPHERE).

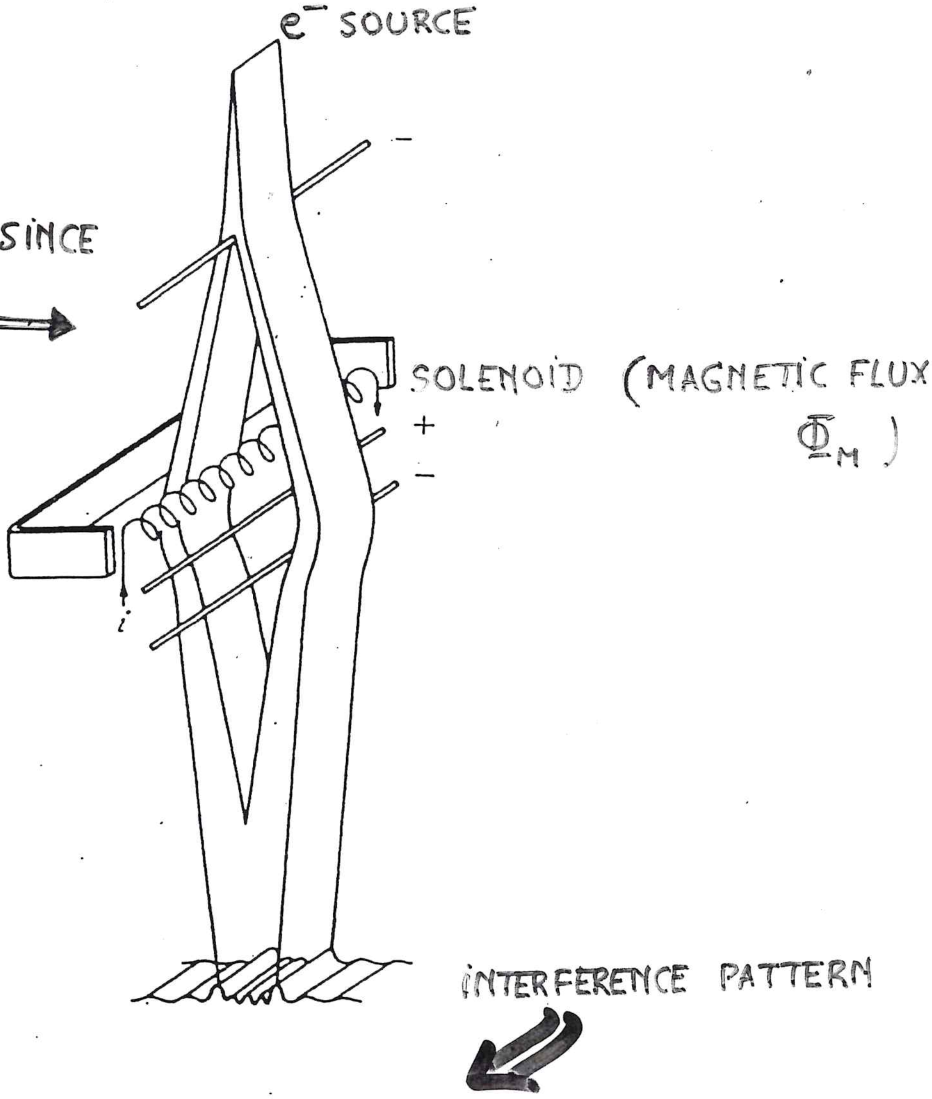
GOING AROUND CLOSED CURVE

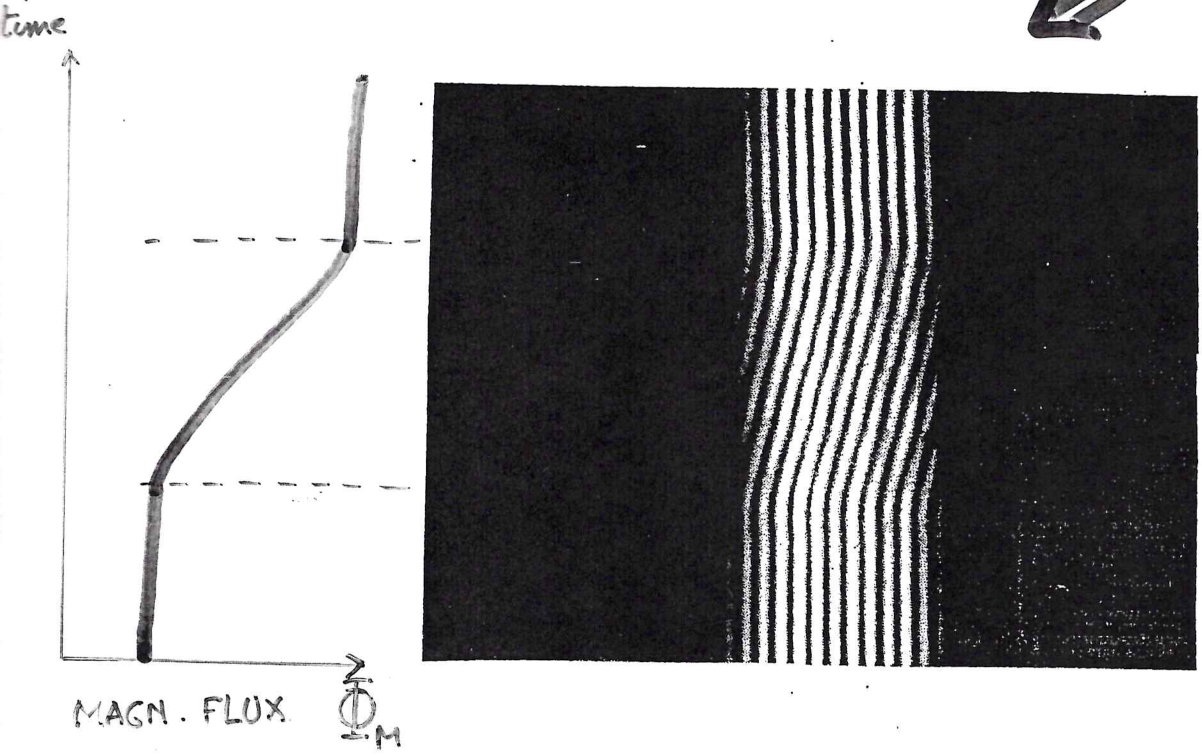


$\Delta\theta = 90°$

EXPERIMENTAL OBSERVATION OF AHARONOV-BOHM EFFECT

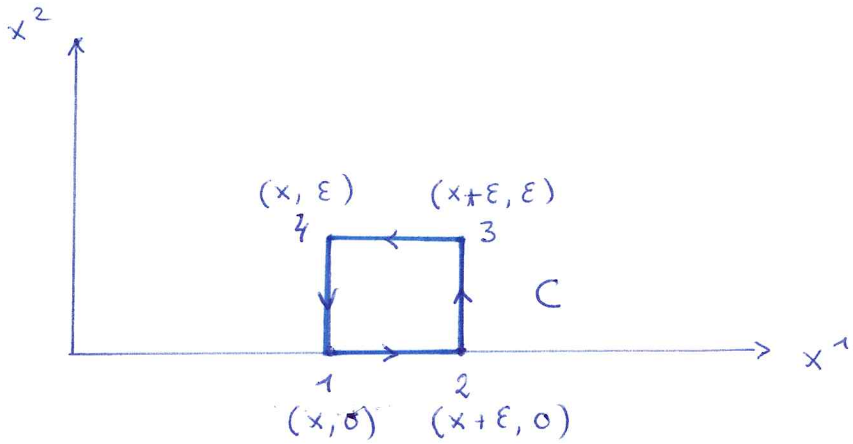
- FIRST EXPERIMENT: CHAMBERS (1960)
- SEVERAL TECHNIQUES SINCE e.g. BAYH (1962) ⟹





• EM FIELD TENSOR

PARALLEL TRANSPORT ALONG INFINITESIMAL CLOSED CURVE C



$$\Phi_C(x) = \exp\left\{-ie \oint_C dx^\mu A_\mu\right\}$$

$$= \Phi(1,4)\,\Phi(4,3)\,\Phi(3,2)\,\Phi(2,1)$$

$$\Phi(2,1) = \exp\left\{-ie \int_1^2 dx^\mu A_\mu\right\}$$

$$= \exp\left\{+ie\,\varepsilon A^1\left(x+\frac{\varepsilon}{2}, 0\right) + O(\varepsilon^2)\right\}$$

$$\Phi(4,3) = \exp\left\{-ie\,\varepsilon A^1\left(x+\frac{\varepsilon}{2}, \varepsilon\right) + O(\varepsilon^2)\right\}$$

$$\Phi(4,3)\,\Phi(2,1) = \exp\left\{-ie\,\varepsilon \underbrace{\left[A^1\left(x+\frac{\varepsilon}{2}, \varepsilon\right) - A^1\left(x+\frac{\varepsilon}{2}, 0\right)\right]}_{\varepsilon \cdot \frac{\partial}{\partial x^2} A^1(x,0) + O(\varepsilon^2)} + O(\varepsilon^3)\right\}$$

$$= \exp\left\{-ie\,\varepsilon^2 \frac{\partial}{\partial x^2} A^1(x,0) + O(\varepsilon^3)\right\}$$

ANALOGOUSLY

$$\Phi(1,4)\,\Phi(3,2) = \exp\left\{+ie\,\varepsilon^2 \frac{\partial}{\partial x^1} A^2(x,0) + O(\varepsilon^3)\right\}$$

$$\therefore \quad \Phi_C(x) = \exp\left\{ ie\, \varepsilon^2 \left[\frac{\partial}{\partial x^1} A^2 - \frac{\partial}{\partial x^2} A^1 \right] + O(\varepsilon^3) \right\}$$

$$= \exp\left\{ -ie\, \underset{\substack{\uparrow \\ \text{SURFACE}}}{\varepsilon^2} \underbrace{\left(\partial^1 A^2 - \partial^2 A^1 \right)}_{\substack{\downarrow \\ F^{12}:\ \text{EM FIELD TENSOR}}} + O(\varepsilon^3) \right\}$$

(cf. MAGN. FLUX IN AHARONOV-BOHM EXP.)

$$\boxed{F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu}$$

$\Rightarrow$ COMPONENTS GIVE $\bar{E}$, $\bar{B}$ FIELDS

$E^i = -F^{0i}$

$B^i = -\frac{1}{2} \varepsilon^{ijk} F^{jk}$

BECAUSE

$$\Phi(y,x) \xrightarrow{U(1)} \exp\{i\theta(y)\}\, \Phi(y,x)\, \exp\{-i\theta(x)\}$$

$$\underset{(x=y)}{\Phi_C(x)} \xrightarrow{U(1)} \Phi_C(x)$$

INVARIANT

$$F^{\mu\nu} \xrightarrow{U(1)} F^{\mu\nu}$$

EM FIELD TENSOR IS GAUGE INVARIANT !

$\rightarrow$ WE CAN ALSO SHOW THAT

$$[D^\mu, D^\nu]_- \psi = [\partial^\mu, \partial^\nu]_- \psi + ie\left([\partial^\mu, A^\nu]_- - [\partial^\nu, A^\mu]_- \right)\psi - e^2 [A^\mu, A^\nu]_- \psi$$

$$= ie\left(\partial^\mu A^\nu - \partial^\nu A^\mu \right) \psi$$

$$\boxed{[D^\mu, D^\nu]_- = ie\, F^{\mu\nu}}$$

• QED LAGRANGIAN → U(1) GAUGE INVARIANCE

$$\mathcal{L}_{QED} = \bar{\psi}\,(i\gamma^\mu D_\mu - m)\,\psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$= \underbrace{\bar{\psi}\,(i\gamma^\mu \partial_\mu - m)\,\psi}_{\mathcal{L}_0} \; \underbrace{- \, e\, \bar{\psi}\gamma^\mu\psi\, A_\mu}_{\mathcal{L}_{INT}} \; \underbrace{- \frac{1}{4} F_{\mu\nu} F^{\mu\nu}}_{\mathcal{L}_{em}}$$

$\mathcal{L}_0$ ↓ DESCRIBES MATTER (SPIN 1/2) IN ABSENCE OF INTERACTIONS

$\mathcal{L}_{INT}$ ↓ DESCRIBES COUPLING OF MATTER TO MATTER

$\mathcal{L}_{em}$ ↓ GIVES MAXWELL'S EQUATIONS IN ABSENCE OF MATTER FIELDS (RADIATION)

$\underbrace{\text{DYNAMICS OF GAUGE FIELD}}$ ↓ MASSLESS!

$\mathcal{L}_{QED}$ is <u>INVARIANT</u> UNDER U(1) GAUGE TF.

$$\boxed{\begin{aligned} \psi(x) &\longrightarrow \exp\{i\,\theta(x)\}\,\psi(x) \\ A^\mu &\longrightarrow A^\mu - \frac{1}{e}\,\partial^\mu\theta \end{aligned}}$$

⇒ NON-ABELIAN GAUGE THEORY, QCD

- QUARKS

$$\mathcal{L}_{\substack{\text{FREE}\\ \text{QUARKS}}} = \bar{q}\,(i\gamma^\mu \partial_\mu - m)\,q$$

QUARKS COME IN 3 COLORS (r, g, b)

$$q = \begin{pmatrix} q_r \\ q_g \\ q_b \end{pmatrix}$$

SU(3)$_c$ GAUGE SYMMETRY : PHYSICS IS INVARIANT UNDER SU(3) LOCAL 'ROTATIONS' OF 3 QUARK COLORS

$$U \in SU(3)_c : \qquad q \rightarrow U q$$

$$U = \exp\{ i\,\theta_a(x)\, t_a \}$$

$t_a = \frac{\lambda_a}{2}$; $a = 1 \ldots 8$

↳ SU(3)$_c$ GENERATORS

WITH

$$[t_a, t_b]_- = i f_{abc}\, t_c$$

- COVARIANT DERIVATIVE

REPLACE $\partial_\mu \Rightarrow$

$$\boxed{D_\mu \equiv \partial_\mu + i g \underbrace{t_a A^a_\mu}_{\equiv A_\mu}}$$

A_μ ⟹ 3×3 MATRIX

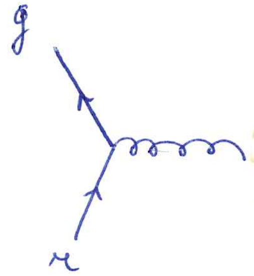
A^a_μ : GAUGE FIELD → 8 GLUONS

g : COUPLING OF QUARKS TO GLUONS

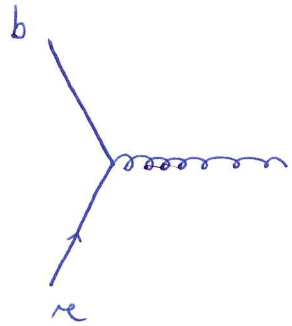
8 POSSIBLE TF OF 3 QUARK COLORS

$\Downarrow$

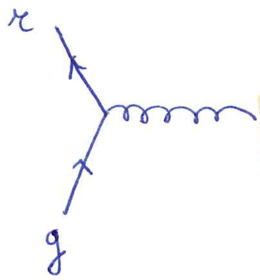
8 GLUONS



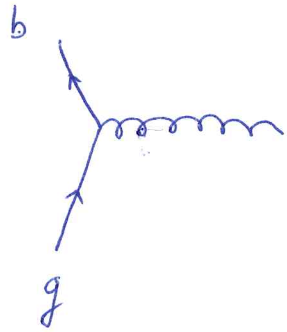
$g_1 = r\bar{g}$



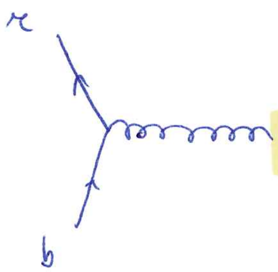
$g_2 = r\bar{b}$



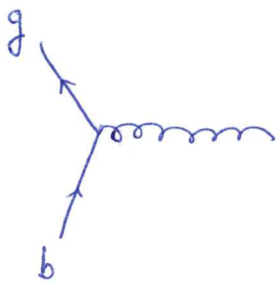
$g_3 = g\bar{r}$



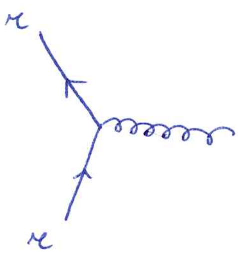
$g_4 = g\bar{b}$



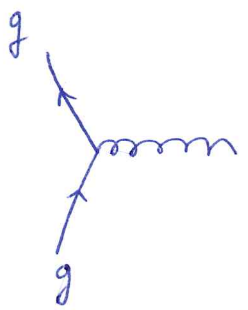
$g_5 = b\bar{r}$



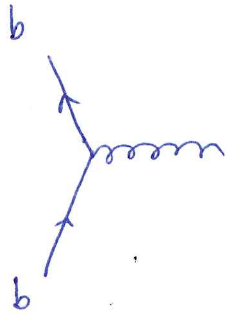
$g_6 = b\bar{g}$



$r\bar{r}$



$g\bar{g}$



$b\bar{b}$

ONLY 2 COMBINATIONS CORRESPOND WITH COLORED GLUON

$\frac{1}{\sqrt{3}}\left(r\bar{r} + g\bar{g} + b\bar{b}\right)$ IS COLOR SINGLET [1]

$g_7 = \frac{1}{\sqrt{2}}\left(r\bar{r} - g\bar{g}\right)$ $\qquad$ $g_8 = \frac{1}{\sqrt{6}}\left(r\bar{r} + g\bar{g} - 2b\bar{b}\right)$

WANT $\bar{q}\gamma^{\mu}D_{\mu}q$ TO BE $SU(3)_c$ GAUGE INVARIANT

BECAUSE $\bar{q} \xrightarrow{SU(3)_c} \bar{q}\,U^{+}$

$$\boxed{D_{\mu}q \xrightarrow{SU(3)_c} U\,(D_{\mu}q)}$$

- HOW DOES A_{μ} TRANSFORM ?

$$D_{\mu}q = \partial_{\mu}q + ig A_{\mu} q$$

$$\xrightarrow{SU(3)_c} \partial_{\mu}(Uq) + ig\,A'_{\mu}U q$$

$$= (\partial_{\mu}U)\,q + U\,(\partial_{\mu}q) + ig\,A'_{\mu}U q$$

ON THE OTHER HAND

$$D_{\mu}q \xrightarrow{SU(3)_c} U\,(D_{\mu}q)$$

$$= U\,(\partial_{\mu}q) + ig\,U A_{\mu} q$$

$$\therefore \quad (\partial_{\mu}U) + ig\,A'_{\mu}U = ig\,U A_{\mu}$$

$$A'_{\mu}U = U A_{\mu} + \frac{i}{g}(\partial_{\mu}U)$$

$$\boxed{A_{\mu} \xrightarrow{SU(3)_c} A'_{\mu} = U A_{\mu} U^{-1} + \frac{i}{g}(\partial_{\mu}U)\,U^{-1}}$$

↳ WE CAN WRITE THIS EQUIVALENTLY USING $U U^{-1} = \mathbb{1}$

$$\downarrow$$

$$(\partial_\mu U) U^{-1} + U (\partial_\mu U^{-1}) = 0$$

$$A_\mu \longrightarrow U A_\mu U^{-1} - \frac{i}{g} U (\partial_\mu U^{-1})$$

↳ TRANSFORMATION LAW FOR INDIVIDUAL FIELDS A_μ^a, $a = 1 \dots 8$

$$U = \exp\{i \theta_a t_a\} \simeq \underbrace{1 + i \theta_a t_a}_{\text{INFINIT. TF.}} + O(\theta^2)$$

$$A_\mu^a t_a \longrightarrow (1 + i \theta_b t_b) A_\mu^c t_c (1 - i \theta_{b'} t_{b'})$$

$$- \frac{i}{g} (1 + i \theta_b t_b)(-i (\partial_\mu \theta_a) t_a)$$

$$\simeq A_\mu^a t_a + i \theta_b A_\mu^c [t_b, t_c] -$$

$$- \frac{1}{g} (\partial_\mu \theta_a) t_a + O(\theta^2)$$

$$= \left[A_\mu^a - f_{abc} \theta_b A_\mu^c - \frac{1}{g} (\partial_\mu \theta_a)\right] t_a + O(\theta^2)$$

$$\therefore \quad \boxed{A_\mu^a \xrightarrow{SU(3)_c} A_\mu^a - \underbrace{\frac{1}{g} (\partial_\mu \theta_a)}_{\text{QED LIKE TERM}} - \underbrace{f_{abc} \theta_b A_\mu^c}_{SU(3)_c \text{ TERM !}}}$$

- EM FIELD TENSOR

→ $$\boxed{[D_\mu, D_\nu]_- \equiv i g \, \mathbb{F}_{\mu\nu}}$$

$$\mathbb{F}_{\mu\nu} \equiv t_a \cdot F^a_{\mu\nu}$$

$$i g \, \mathbb{F}_{\mu\nu} = [\partial_\mu + i g A_\mu , \partial_\nu + i g A_\nu]_-$$

$$= i g \left(\partial_\mu A_\nu - \partial_\nu A_\mu \right) + (i g)^2 \, [A_\mu, A_\nu]_-$$

$$\Downarrow$$

$$\boxed{\mathbb{F}_{\mu\nu} = \underbrace{\partial_\mu A_\nu - \partial_\nu A_\mu}_{\text{cfr. ABELIAN CASE}} + i g \underbrace{[A_\mu, A_\nu]_-}_{\text{SELF-INTERACTIONS BETWEEN } g \text{; NON-ABELIAN TERM}}}$$

$$\boxed{F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f_{abc} A^b_\mu A^c_\nu}$$

→ TRANSFORMATION LAW

$$[D_\mu, D_\nu]_- \psi \to U [D_\mu, D_\nu]_- U^\dagger (U\psi)$$

$$= i g \, \mathbb{F}'_{\mu\nu} (U\psi)$$

$$\boxed{\mathbb{F}^{\mu\nu} \to \mathbb{F}'^{\mu\nu} = U \mathbb{F}^{\mu\nu} U^\dagger}$$

• GAUGE - FIELD TERM IN $\mathcal{L}$

WE NEED A GAUGE INV. TERM

→ IN CASE OF $U(1)$ $\quad \mathcal{L}_{GAUGE} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$

$$F_{\mu\nu} \to F_{\mu\nu}$$

→ IN CASE OF $SU(3)_c$: $\mathbb{F}_{\mu\nu} \to U \mathbb{F}_{\mu\nu} U^{+}$

GAUGE INV. TERM , BILINEAR IN $\mathbb{F}_{\mu\nu}$:

$$\mathrm{Tr}\left(\mathbb{F}_{\mu\nu} \mathbb{F}^{\mu\nu}\right) = F^{a}_{\mu\nu} F^{b\,\mu\nu} \underbrace{\mathrm{Tr}\left(t_a t_b\right)}_{\frac{1}{2}\delta_{ab}}$$

$$\boxed{\mathcal{L}_{YM} = -\frac{1}{2} \mathrm{Tr}\left\{\mathbb{F}_{\mu\nu} \mathbb{F}^{\mu\nu}\right\} = -\frac{1}{4} \underbrace{F^{a\mu\nu} F^{a}_{\mu\nu}}_{\text{SUM OVER 8 GLUONS}}}$$

↑
YANG - MILLS

• BECAUSE $\mathbb{F}^{\mu\nu}$ HAS $[A^{\mu}, A^{\nu}]_-$ TERM

$\mathcal{L}_{YM}$ HAS CUBIC & QUARTIC TERMS IN $\mathbb{A}$

↳ SELF - INTERACTIONS AMONG GLUONS

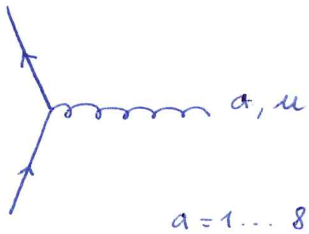
FEATURE OF NON - ABELIAN GAUGE THEORY

⇒ FEYNMAN RULES IN QCD

$$\boxed{\mathcal{L}_{QCD} = \bar{q}\,(i\gamma^\mu \partial_\mu - m)\,q - g\,\bar{q}\,\gamma^\mu \frac{\lambda_a}{2}\,q\,A^a_\mu - \frac{1}{4} F^{a\mu\nu} F^a_{\mu\nu}}$$

- FEYNMAN RULE FOR $q\bar{q}g$ VERTEX

FEYNMAN RULE
⇕
S-MATRIX "$S = i\mathcal{L}$"



$a = 1 \dots 8$

$\mu = 0, 1, 2, 3$

$$\boxed{-i g\, \gamma^\mu \frac{\lambda_a}{2}}$$

- $\mathcal{L}_{YM} = -\frac{1}{4} F^{a\mu\nu} F^a_{\mu\nu}$

$$= -\frac{1}{4}\left(\partial^\mu A^{a\nu} - \partial^\nu A^{a\mu} - g f_{abc} A^{b\mu} A^{c\nu}\right)$$

$$\cdot \left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f_{abc} A^b_\mu A^c_\nu\right)$$

$$= -\frac{1}{4}\left(\partial^\mu A^{a\nu} - \partial^\nu A^{a\mu}\right)\left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu\right) \quad \Rightarrow \text{FREE GLUONS}$$

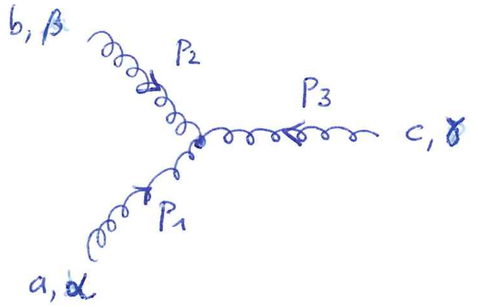
$$+ g f_{abc} \left(\partial^\mu A^{a\nu}\right) A^b_\mu A^c_\nu \quad \Rightarrow 3g \text{ INTERACTION}$$

$$- \frac{1}{4} g^2 f_{abc} f_{ade} A^{b\mu} A^{c\nu} A^d_\mu A^e_\nu \quad \Rightarrow 4g \text{ INTERACTION}$$

<u>NOTE</u> : SAME COUPLING g APPEARS IN $3g$ AND $4g$ INTERACTIONS

FEYNMAN RULE FOR 3g VERTEX

$$\mathcal{L}_{3g} = g\, f_{abc} \left(\partial^{\mu} A^{a\nu} \right) A^{b}_{\mu} A^{c}_{\nu}$$



$$\begin{aligned} g\, f_{abc} \Big\{ & (p_1)^{\mu}\, g^{\alpha\nu} \left(g^{\beta}_{\mu} g^{\gamma}_{\nu} - g^{\beta}_{\nu} g^{\gamma}_{\mu} \right) \\ & + (p_2)^{\mu}\, g^{\beta\nu} \left(g^{\gamma}_{\mu} g^{\alpha}_{\nu} - g^{\gamma}_{\nu} g^{\alpha}_{\mu} \right) \\ & + (p_3)^{\mu}\, g^{\gamma\nu} \left(g^{\alpha}_{\mu} g^{\beta}_{\nu} - g^{\alpha}_{\nu} g^{\beta}_{\mu} \right) \Big\} \end{aligned}$$

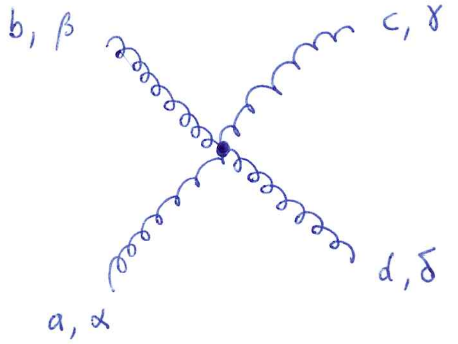
$\Downarrow$

FEYNMAN ROLE

$$\boxed{\begin{aligned} - g\, f_{abc} \Big\{ & g^{\alpha\beta} \left(p_1 - p_2 \right)^{\gamma} \\ & + g^{\beta\gamma} \left(p_2 - p_3 \right)^{\alpha} \\ & + g^{\gamma\alpha} \left(p_3 - p_1 \right)^{\beta} \Big\} \\ \equiv - g\, f_{abc}\, & V^{\alpha\beta\gamma} \left(p_1, p_2, p_3 \right) \end{aligned}}$$

- FEYNMAN RULE FOR 4g VERTEX

$$\mathcal{L}_{4g} = -\frac{1}{4} g^2 f_{ea'b'} f_{ec'd'} A^{a'\mu} A^{b'\nu} A^{c'}_{\mu} A^{d'}_{\nu}$$



$$-\,4 \cdot \frac{i}{4} g^2 \Big\{ \underline{f_{eab} f_{ecd}} \left(g^{\mu}_{\alpha} g^{\nu}_{\beta} g_{\gamma\mu} g_{\delta\nu} \right)$$

↑ 4 POSSIBLE WAYS TO CHOOSE FIRST GLUON

$$+ \underline{f_{eab} f_{edc}} \left(g^{\mu}_{\alpha} g^{\nu}_{\beta} g_{\delta\mu} g_{\gamma\nu} \right)$$

$$+ \underline{f_{eac} f_{ebd}} \left(g^{\mu}_{\alpha} g^{\nu}_{\gamma} g_{\beta\mu} g_{\delta\nu} \right)$$

$$+ \underline{f_{eac} f_{edb}} \left(g^{\mu}_{\alpha} g^{\nu}_{\gamma} g_{\delta\mu} g_{\beta\nu} \right)$$

$$+ \underline{f_{ead} f_{ebc}} \left(g^{\mu}_{\alpha} g^{\nu}_{\delta} g_{\beta\mu} g_{\gamma\nu} \right)$$

$$+ \underline{f_{ead} f_{ecb}} \left(g^{\mu}_{\alpha} g^{\nu}_{\delta} g_{\gamma\mu} g_{\beta\nu} \right) \Big\}$$

↓

$$\boxed{\begin{aligned} & -i g^2 \Big\{ f_{abe} f_{cde} \left(g_{\alpha\gamma} g_{\beta\delta} - g_{\alpha\delta} g_{\beta\gamma} \right) \\ & \quad + f_{ace} f_{bde} \left(g_{\alpha\beta} g_{\gamma\delta} - g_{\alpha\delta} g_{\beta\gamma} \right) \\ & \quad + f_{ade} f_{bce} \left(g_{\alpha\beta} g_{\gamma\delta} - g_{\alpha\gamma} g_{\beta\delta} \right) \Big\} \\ & \equiv -i g^2 W^{abcd}_{\alpha\beta\gamma\delta} \longrightarrow \text{FULLY SYMMETRIC UNDER INTERCHANGE OF 2 INDICES} \end{aligned}}$$