

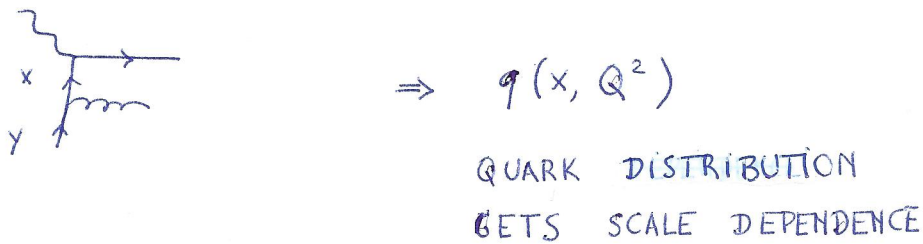
SCALING VIOLATIONS IN DIS, EVOLUTION OF PARTON DISTRIBUTIONS

* INTRODUCTION

→ DIS IN LO IN α_s



→ DIS IN NLO IN α_s



Q^2 DEPENDENCE OF PARTON DISTRIBUTION GOVERNED BY EVOLUTION EQUATION

2 TYPES OF EVOLUTION EQUATIONS:

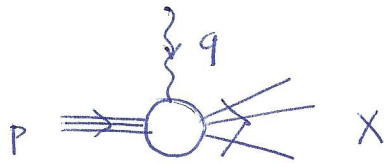
→ FLAVOR NON-SINGLET

$$q_v = q - \bar{q}$$

→ FLAVOR SINGLET ($SU(3)_F$ SINGLET)

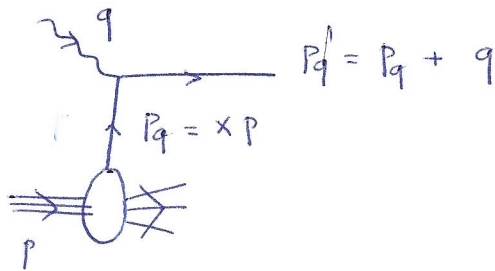
$$\left. \begin{array}{l} \frac{1}{3} (u + d + s) \\ g \text{ (GLUON)} \end{array} \right\} \text{QUARK SINGLET \& GLUON MIX.}$$

* DIS TENSOR : GENERAL



$$W^{\mu\nu} = \frac{1}{2\pi} \sum_X (2\pi)^4 \delta^4(P + q - P_X) \langle P | J^{\mu+}(0) | X \rangle \langle X | J^{\nu}(0) | P \rangle$$

* DIS TENSOR : PARTON MODEL



$$W_{\text{PARTON}}^{\mu\nu} = \sum_q e_q^2 \int_0^1 \frac{dx}{x} q(x) \omega_{q \rightarrow q}^{\mu\nu}$$

INITIAL FLU. ↑
 TENSOR FOR SCATTERING OFF QUARK

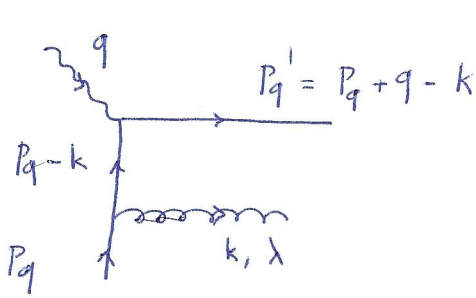
$$\frac{1}{2E_q} = \frac{1}{x(2E)} \quad (E = \sqrt{P^2 + M^2})$$

UNPOL SCATTERING

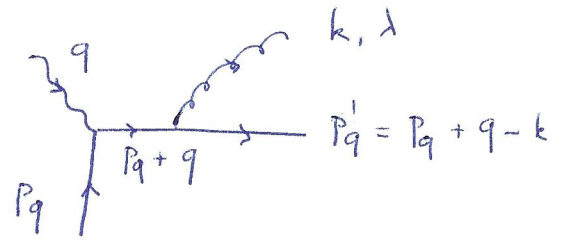
$$\omega_{q \rightarrow q}^{\mu\nu} = \frac{1}{2\pi} \frac{1}{2} \sum_{s, s'} \int \frac{d^3 P_q}{(2\pi)^3 2E_q} (2\pi)^4 \delta^4(P_q + q - P_q') \bar{U}_{q \rightarrow q}^{\mu*} J_{q \rightarrow q}^{\nu}$$

$$J_{q \rightarrow q}^{\nu} = \bar{U}(P_q', s') \gamma^{\nu} U(P_q, s)$$

* DIS TENSOR : ONE - GLUON EMISSION



(a)



(b)

$$W_{q \rightarrow qg}^{\mu\nu} = \frac{1}{2\pi} \underbrace{\frac{1}{2} \sum_{ss'}}_{\text{UNPOL.}} \sum_{\lambda} \int \frac{d^3 \vec{k}}{(2\pi)^3 2|\vec{k}|} \int \frac{d^3 \vec{p}_q'}{(2\pi)^3 2E_q'} (2\pi)^4 \delta^4(p_q + q - p_q' - k')$$

$$J_{q \rightarrow qg}^{\mu*} \quad J_{q \rightarrow qg}^{\nu}$$

$$J_{q \rightarrow qg}^{\nu}(a) = \bar{U}(p_q + q - k, s') \gamma^{\nu} \frac{i(p_q - k)}{(p_q - k)^2} \left[-ig \gamma^{\beta} \frac{\lambda_a}{z} \right] U(p_q, s) \cdot \epsilon_{\beta}^*(k, \lambda)$$

$$J_{q \rightarrow qg}^{\nu}(b) = \bar{U}(p_q + q - k, s') \left[-ig \gamma^{\beta} \frac{\lambda_a}{z} \right] \frac{i(p_q + q)}{(p_q + q)^2} \gamma^{\nu} U(p_q, s) \cdot \epsilon_{\beta}^*(k, \lambda)$$

$$\hookrightarrow \int \frac{d^3 \vec{k}}{(2\pi)^3 2|\vec{k}|} \dots = \int \frac{d^4 k}{(2\pi)^3} \delta_+(k^2) \dots \quad \delta_+(k^2) \equiv \delta(k^2) \Theta(k^0)$$

$$\hookrightarrow \int \frac{d^3 \vec{p}_q'}{(2\pi)^3 2E_q'} \dots = \int \frac{d^4 p_q'}{(2\pi)^3} \delta_+(p_q'^2 - m_q^2) \dots$$

↳ COLOR FACTOR SU(3)

$$C_F = \frac{1}{3} \sum_{i=1,2,3} \sum_{a=1}^8 \left(\frac{\lambda_a}{2} \cdot \frac{\lambda_a}{2} \right) = \frac{1}{3} \cdot 4 = \frac{4}{3}$$

↑
AVERAGE
OVER 3

INITIAL QUARK COLORS

↑
SUM OVER 8 GLUONS

$$\hookrightarrow w_{q \rightarrow qg}^{\mu\nu}(a) = \frac{1}{2} \cdot \frac{1}{(2\pi)^3} \cdot g^2 \cdot C_F$$

$$\cdot \int \frac{d^3 \vec{k}}{2|\vec{k}|} \delta_+((p_q + q - k)^2) \left[\sum_{\lambda} \epsilon_{\alpha}(k, \lambda) \epsilon_{\beta}^*(k, \lambda) \right]$$

$$\cdot \frac{1}{(p_q - k)^4} \cdot \text{Tr} \left\{ \gamma^{\alpha}(p_q - k) \gamma^{\mu}(p_q + q - k) \gamma^{\nu}(p_q - k) \gamma^{\beta} p_q \right\}$$

$$\hookrightarrow w_{q \rightarrow qg}^{\mu\nu}(b) = \frac{1}{2} \cdot \frac{1}{(2\pi)^3} \cdot g^2 \cdot C_F$$

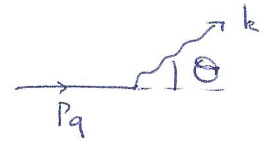
$$\cdot \int \frac{d^3 \vec{k}}{2|\vec{k}|} \delta_+((p_q + q - k)^2) \left[\sum_{\lambda} \epsilon_{\alpha}(k, \lambda) \epsilon_{\beta}^*(k, \lambda) \right]$$

$$\cdot \frac{1}{(p_q + q)^4} \cdot \text{Tr} \left\{ \gamma^{\mu}(p_q + q) \gamma^{\alpha}(p_q + q - k) \gamma^{\beta}(p_q + q) \gamma^{\nu} p_q \right\}$$

↳ DIAGRAM (a) HAS SINGULARITY

WHEN $(P_q - k)^2 = 0$

$$(P_q - k)^2 = P_q^2 - 2|k| (P_q^0 - |\vec{P}_q| \cos \theta)$$



$$\approx -2|k| |\vec{P}_q| (1 - \cos \theta)$$

FOR
MASSLESS
QUARKS

|| $\cos \theta = 1 \leftrightarrow$ COLLINEAR DIVERGENCE

DIV. WHEN GLUON IS EMITTED IN DIR. OF QUARK

- REGULARIZE BY PUTTING QUARK OFF-SHELL

$$\text{i.e. } P_q^2 = -\mu^2$$

- DIVERGENT TERM IN $\mu \rightarrow 0$ LIMIT

↳ LOGARITHMIC CORRECTION $\ln(Q^2/\mu^2)$

↳ DIAGRAM (b) HAS NO SINGULARITY OVER GLUON INTEGRATION RANGE

$$(P_q + q)^2 \text{ IS INDEPENDENT OF GLUON MOMENTUM } k$$

↳ CHOOSE AXIAL GAUGE ($A^+ = 0$) $\Leftrightarrow A^0 + A^3 = 0$
 p HAS LARGE POSITIVE Z-COMP.

$m \cdot A = 0$ WITH $m^\mu = \frac{1}{\sqrt{2}} (1, 0, 0, -1)$

$$\sum_{\lambda=\pm 1} \epsilon_\alpha(k, \lambda) \epsilon_\beta^*(k, \lambda) = -g_{\alpha\beta} + \frac{k_\alpha m_\beta + k_\beta m_\alpha}{k \cdot m}$$

$m \cdot \epsilon = 0$

$m^2 = 0$ (LIGHTLIKE VECTOR)

↳ $w_{q \rightarrow qg(a)}^{\mu\nu} = \frac{1}{2} \cdot \frac{1}{(2\pi)^3} g^2 \frac{4}{3}$

$\cdot \int \frac{d^3k}{2|k|} \delta_+((p_q + q - k)^2) \cdot \frac{1}{(p_q - k)^4}$

$\cdot \left\{ 2 \text{Tr} \left\{ \gamma^\mu (p_q + q - k) \gamma^\nu \underbrace{(p_q - k) p_q (p_q - k)}_{2(k \cdot p_q) k} \right\} \right.$
 $+ \frac{1}{k \cdot m} \text{Tr} \left\{ \gamma^\mu (p_q + q - k) \gamma^\nu (p_q - k) \underbrace{m p_q k (p_q - k)}_{2k \cdot p_q p_q} \right\}$
 $\left. + \frac{1}{k \cdot m} \text{Tr} \left\{ \gamma^\mu (p_q + q - k) \gamma^\nu \underbrace{(p_q - k) k p_q m (p_q - k)}_{2k \cdot p_q p_q} \right\} \right\}$

$w_{q \rightarrow qg(a)}^{\mu\nu} = \frac{1}{2} \cdot \frac{1}{(2\pi)^3} g^2 \frac{4}{3}$

$\cdot \int \frac{d^3k}{2|k|} \delta_+((p_q + q - k)^2) \cdot \frac{1}{(p_q - k)^4} \cdot (-2) (p_q - k)^2$

$\cdot \text{Tr} \left\{ \gamma^\mu (p_q + q - k) \gamma^\nu \left[k + \frac{1}{k \cdot m} \left(m \cdot (p_q - k) p_q + m \cdot p_q (p_q - k) + k \cdot p_q m \right) \right] \right\}$

↳ FOR LEADING LOG TERM → DUE TO $\frac{1}{(P_q - k)^2}$

KEEP
ONE CAN TAKE $\Theta = 0$ EVERYWHERE EXCEPT IN DENOMINATOR

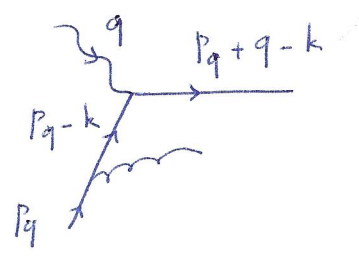
CHOOSE FRAME

$$q = Q(0, 0, 0, -1)$$

$$P_q = \frac{Q}{2x}(1, 0, 0, 1) \rightarrow P_q^2 = 0$$

$$P_q \cdot q = \frac{Q^2}{2x}$$

$$k \approx k_{\Theta=0} = |\bar{k}|(1, 0, 0, -1)$$



$$(P_q - k)^2 = 0$$

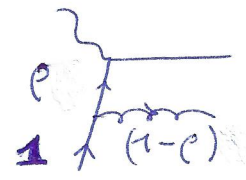
$$\delta_+((P_q + q - k)^2)$$

$$= \delta_+ \left(\frac{(P_q - k)^2 - Q^2 + 2q \cdot (P_q - k)}{0} \right)$$

$$= \frac{1}{2P_q \cdot q} \delta \left(x - \frac{q \cdot (P_q - k)}{q \cdot P_q} \right)$$

$$e \equiv \frac{q \cdot (P_q - k)}{q \cdot P_q} = 1 - \frac{|\bar{k}| 2x}{Q}$$

$$= \frac{1}{2P_q \cdot q} \delta(x - e)$$



$$\therefore k_{\Theta=0}^u = (1 - e) P_q^u$$

$$\hookrightarrow w_{q \rightarrow qg}^{\mu\nu} = \frac{1}{2} \cdot \frac{g^2}{(2\pi)^3} \cdot \frac{4}{3}$$

$$\cdot \frac{2\pi \int_{-1}^1 d\cos\theta \int_0^\infty d|k| \frac{|k|}{4P_q \cdot q} \cdot \frac{(-2) \delta(x-p)}{2|k| \frac{Q}{2x} \cos\theta - (\mu^2 + 2|k| \frac{Q}{2x})}}$$

$$\left[\begin{aligned} & \cdot \left\{ (1-p) \text{Tr} \left\{ \gamma^\mu (p P_q + q) \gamma^\nu P_q \right\} \right. \\ & \left. + \frac{1}{(1-p)} \text{Tr} \left\{ \gamma^\mu (p P_q + q) \gamma^\nu (2p) P_q \right\} \right\} \end{aligned} \right]$$

$$\llcorner \frac{1+p^2}{1-p} \cdot 4 \left\{ (p P_q + q)^\mu P_q^\nu + (p P_q + q)^\nu P_q^\mu - (p P_q + q) \cdot P_q g^{\mu\nu} \right\}$$

⇓

$$w_{q \rightarrow qg}^{\mu\nu} = - \frac{g^2}{(2\pi)^2} \cdot \frac{4}{3} \int_0^\infty d|k| \frac{x}{Q} \frac{\delta(x-p)}{4P_q \cdot q} \ln \left| \frac{2|k| \frac{Q}{2x} - (\mu^2 + 2|k| \frac{Q}{2x})}{2|k| \frac{Q}{2x} + (\mu^2 + 2|k| \frac{Q}{2x})} \right|$$

$$\cdot \frac{1+p^2}{1-p} \cdot 4 (P_q \cdot q) \left\{ \left(-g^{\mu\nu} - \frac{q^\mu q^\nu}{2p(P_q \cdot q)} \right) + \frac{2p}{P_q \cdot q} \left(p^\mu + \frac{q^\mu}{2p} \right) \left(p^\nu + \frac{q^\nu}{2p} \right) \right\}$$

↓

$$d|k| = - \frac{Q}{2x} dp \quad \begin{aligned} p=0 & \Rightarrow |k| = \infty \\ p=1 & \Rightarrow |k| = 0 \end{aligned}$$

$$w^{\mu\nu}_{q \rightarrow qg(a)} = -\frac{g^2}{(2\pi)^2} \frac{4}{3} \int_0^1 dp \frac{1+p^2}{1-p} \delta(x-p) \ln \left| \frac{\mu^2}{\frac{Q^2}{2x^2} (1-p)^2 + \mu^2} \right|$$

$$\cdot \left\{ (-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}) \right.$$

$$\left. + \frac{2x}{p_q \cdot q} \left(p_q^\mu - \frac{p_q \cdot q}{q^2} q^\mu \right) \left(p_q^\nu - \frac{p_q \cdot q}{q^2} q^\nu \right) \right\}$$

$$\downarrow \ln \left[\frac{Q^2}{2x^2} (1-x)^2 + \mu^2 \right] / \mu^2 \approx \ln \frac{Q^2}{\mu^2} + \dots$$

KEEP ONLY
LEADING LOG
TERM IF
 $Q^2 \gg \mu^2$

$$\approx \frac{1}{4} \cdot \frac{4}{3} \cdot \frac{\alpha_s}{\pi} \int_0^1 dp \frac{1+p^2}{1-p} \delta(x-p) \ln \left(\frac{Q^2}{\mu^2} \right)$$

$$\cdot 2 \left\{ (-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}) + \frac{2x}{p_q \cdot q} \left(p_q^\mu - \frac{p_q \cdot q}{q^2} q^\mu \right) \left(p_q^\nu - \frac{p_q \cdot q}{q^2} q^\nu \right) \right\}$$

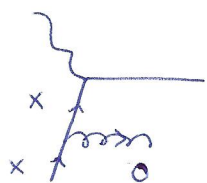
\Downarrow

$$f_2(x, Q^2) \underset{\text{LEADING LOG}}{\approx} \frac{1}{2} \frac{4}{3} \frac{\alpha_s}{\pi} \int_0^1 dp \frac{1+p^2}{1-p} p \delta(x-p) \ln \left(\frac{Q^2}{\mu^2} \right)$$

$$\equiv 2x f_1(x, Q^2)$$

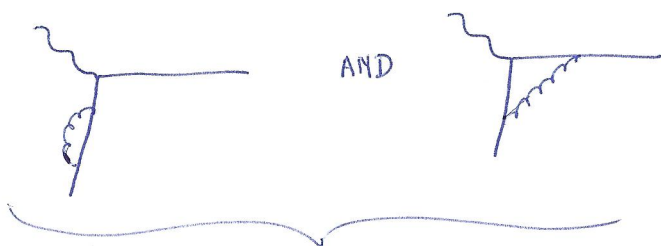
↳ SOFT-GLUON INFRARED SINGULARITY

$\rho \rightarrow 1$



$\frac{1}{1-\rho}$ SINGULAR

CANCELLED BY



IN SAHL HAS SIMILAR STRUCTURE

BUT $\frac{1+\rho^2}{1-\rho} \rightarrow \lambda \delta(1-\rho)$ (λ IS CONSTANT)

$$f_2(x, Q^2)_{LL} = \frac{4}{3} \cdot \frac{\alpha_s}{2\pi} \ln \frac{Q^2}{u^2} \int_0^1 d\rho \rho \delta(x-\rho) \cdot \left[\frac{1+\rho^2}{(1-\rho)_+} + \lambda \delta(1-\rho) \right]$$

WE INTRODUCED PRINCIPLE VALUE (i.e. SUBTRACT δ -FUNCTION)

$$\int_0^1 d\rho \frac{f(\rho)}{(1-\rho)_+} \equiv \int_0^1 d\rho \frac{f(\rho) - f(1)}{(1-\rho)}$$

↳ CORRESPONDS WITH FINITE PART

NOTE

$$\frac{1}{(1-\rho)_+} = \lim_{\epsilon \rightarrow 0^+} \left\{ \frac{1}{1-\rho} \theta(1-\rho-\epsilon) - \delta(1-\rho) \int_0^{1-\epsilon} d\rho' \frac{1}{1-\rho'} \right\}$$

BY CHANGING $\frac{1}{1-\rho} \rightarrow \frac{1}{(1-\rho)_+}$ A PIECE OF δ -FUNCTION IS BROUGHT IN SINGULAR TERM TO CANCEL DIVERGENCE

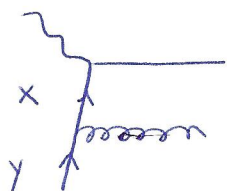
* LEADING LOG CORRECTION

TO VALENCE QUARK DISTRIBUTION (NON-SINGLET)

(NS)



$$q_v(x)$$



$$q_v(x, Q^2)$$

LOGARITHMIC $\cdot \ln \frac{Q^2}{\mu^2}$

QUARK STRUCK BY PHOTON HAS FRACTION $z = \frac{x}{y}$ OF INITIAL QUARK MOMENTUM

$$\hookrightarrow f_2^{NS} \left(\frac{x}{y}, Q^2 \right) \underset{LL}{=} \frac{4}{3} \frac{\alpha_s}{2\pi} \cdot \ln \frac{Q^2}{\mu^2} \cdot \frac{x}{y} \left[\frac{1+z^2}{(1-z)_+} + \lambda \delta(1-z) \right] \quad z = \frac{x}{y}$$

$$f_2^{NS} \left(\frac{x}{y}, Q^2 \right) = \frac{\alpha_s}{2\pi} \cdot \ln \frac{Q^2}{\mu^2} \cdot \frac{x}{y} \cdot P_{q \rightarrow qg}^{NS} \left(\frac{x}{y} \right)$$

NON-SINGLET SPLITTING FUNCTION

$$P_{q \rightarrow qg}^{NS}(z) = \frac{4}{3} \cdot \left[\frac{1+z^2}{(1-z)_+} + \lambda \delta(1-z) \right]$$

↳ $\delta q_v(x, Q^2)$: CORRECTION TO VALENCE QUARK DISTR.
DUE TO GLUON EMISSION

$\delta q_v(x, Q^2)$ FROM $\frac{1}{x} F_2(x, Q^2)$

$$\delta q_v(x, Q^2) = \frac{\alpha_s}{2\pi} \cdot \ln \frac{Q^2}{\mu^2} \int_x^1 \frac{dy}{y} P_{q \rightarrow qg}^{NS} \left(\frac{x}{y} \right) \cdot q_v(y, Q^2)$$

$q_v(y, Q^2)$: QUARK DENSITY \leadsto # QUARKS WITH MOMENTUM FRACTION y IN NUCLEON

$P_{q \rightarrow qg} \left(\frac{x}{y} \right)$: PROBABILITY THAT QUARK WITH MOM. FRAC y SPLITS INTO q & g ,
QUARK ENDS UP WITH MOM. FRAC. x ($x \leq y$)

↳ LEADING LOG EVOLUTION (DGLAP EQUATION)

DOKSHITZER
GRIBOV
LIPATOV
ALTARELLI
PARISI

$$q_v(x, Q^2) = q_v(x) + \delta q_v(x, Q^2)$$

⇓

$$\frac{\partial}{\partial \ln Q^2} q_v(x, Q^2) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} P_{q \rightarrow qg}^{NS} \left(\frac{x}{y} \right) q_v(y, Q^2)$$

DGLAP EQUATION

WITH

$$P_{q \rightarrow qg}^{NS}(z) = \frac{1+z^2}{(1-z)_+} + \lambda \delta(1-z)$$

DETERMINE λ

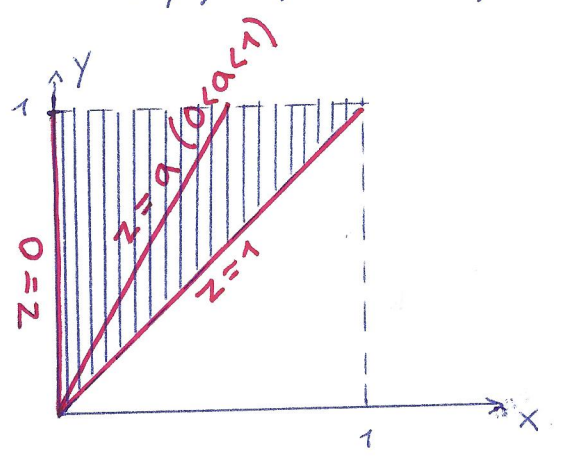
TOTAL NUMBER OF VALENCE QUARKS OF A GIVEN FLAVOR IS INDEPENDENT OF RESOLUTION SCALE Q

$$\frac{\partial}{\partial \ln Q^2} \int_0^1 dx q_v(x, Q^2) = 0$$

⇓

$$0 = \frac{\alpha_s}{2\pi} \int_0^1 dx \int_x^1 \frac{dy}{y} P^{NS}\left(\frac{x}{y}\right) g_\sigma(y, Q^2)$$

$$\downarrow \quad z = \frac{x}{y}$$



$$0 = \frac{\alpha_s}{2\pi} \int_0^1 dy \int_0^1 dz P^{NS}(z) g_\sigma(y, Q^2)$$

$$= \frac{\alpha_s}{2\pi} \left(\int_0^1 dy g_\sigma(y, Q^2) \right) \cdot \left(\int_0^1 dz P^{NS}(z) \right)$$

⇓

$$\int_0^1 dz P^{NS}(z) = 0$$

$$\int_0^1 dz \left\{ \frac{1+z^2}{(1-z)_+} + \lambda \delta(1-z) \right\} = 0$$

$$\int_0^1 dz \left(\frac{1+z^2 - 2}{1-z} \right) + \lambda = 0 \Rightarrow \boxed{\lambda = \frac{3}{2}}$$

↳ STRATEGY TO SOLVE DGLAP EQUATION
(INTEGRO-DIFFERENTIAL EQ.)

- START WITH ANSATZ OF $q_v(x, Q^2)$
AT SOME LOW INPUT SCALE Q_0^2 (e.g. $Q_0^2 = 1 \text{ GeV}^2$)
TYPICAL

$$q_v(x, Q_0^2) = A x^a (1-x)^b \{ 1 + c\sqrt{x} + d x \}$$

A DETERMINED FROM NORMALIZATION

$$\int_0^1 dx u_v(x, Q^2) = 2$$

$$\int_0^1 dx d_v(x, Q^2) = 1$$

- USE DGLAP EQUATION TO CALCULATE $q_v(x, Q^2)$ AT LARGER Q^2

Q^2 EVOLUTION : LOGARITHMIC

- FOR $SU(3)_F$ SINGLET DISTRIBUTION

$\frac{1}{3} (u + d + s)$ CAN MIX WITH $g(x, Q^2)$
GLUON DISTR.

