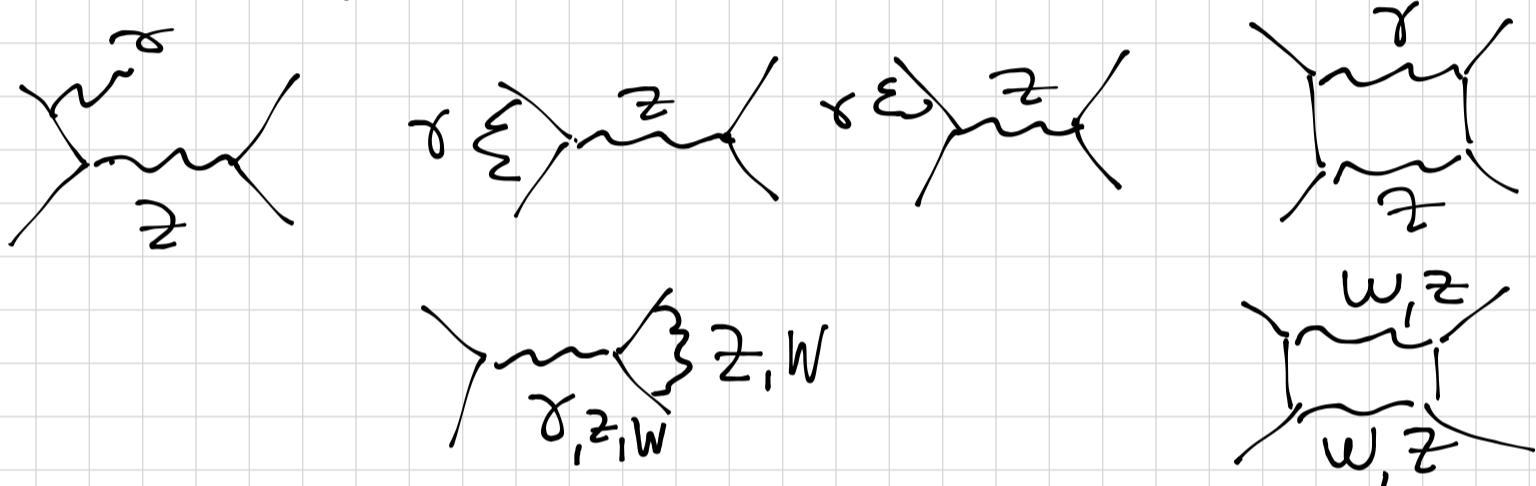


# Lecture 20

The oblique corrections are universal  
(and specific for SM)

Other corrections depend on kinematics  
(process-specific)



Renormalized  
W-mass

$$M_W = \frac{\sqrt{\pi \alpha} / \sqrt{2} G_F}{\hat{s}_Z (1 - \Delta \hat{r}_W)^{1/2}}$$

$$\text{where } \Delta \hat{r}_W \approx 1 - \frac{\alpha}{2 \cdot (M_Z)} \simeq 0.06629(7)$$

with other small corr.

$$\Delta \hat{F}_W = 0.06918(7)$$

=

Renormalized  
Z-mass

$$M_Z = \frac{M_W}{\hat{p}^{1/2} \hat{c}_Z}$$

$$\hat{p} \approx 1 + \rho_t \quad \rho_t = \frac{3 G_F m_t^2}{8 \sqrt{2} \pi^2} \simeq 0.00537 \left( \frac{m_t}{172.89} \right)^2$$

To compare: in on-shell scheme

$$M_W = \frac{\sqrt{T_1 \alpha / \sqrt{2} G_F}}{S_W (1 - \Delta r)^{1/2}}$$

$$\Pi_Z = \frac{M_W}{C_W}$$

$$\Delta r = 1 - \frac{\alpha}{\alpha(M_Z)} - c_f g^2 \Pi_W \rho_t = 0.03652(22)$$

Enhanced sensitivity to  $m_t$ ; lower precision

$$\hat{M}_Z^2 = M_Z^2 + \text{Re}(\Pi_{ZZ}(M_Z^2))$$

~~in~~<sup>Z</sup> ~~W~~  $\Pi_{ZZ}$

$$\hat{M}_W^2 = M_W^2 + \text{Re}(\Pi_{WW}(M_Z^2))$$

~~in~~<sup>W</sup> ~~W~~  $\Pi_{WW}$

The photon acquires no mass; but coupling is renormalized

Total c.s. for  $e^+ e^- \rightarrow \mu^+ \mu^-$

$$G = \frac{\hat{e}^4(M_Z)}{12\pi M_Z^2} = \frac{e^4}{12\pi [M_Z^2 - \Pi_{gg}(M_Z^2)]}$$

$$\Rightarrow \hat{\alpha}(M_Z^2) = \frac{\alpha}{1 - \Pi_{gg}(M_Z^2)/M_Z^2} = \alpha \left(1 + \frac{\Pi_{gg}(M_Z^2)}{M_Z^2} + \dots\right)$$

Fermi constant

$$\frac{\hat{G}_F}{\sqrt{2}} = -\frac{g^2}{8} \frac{1}{p^2 - M_W^2 - \Pi_{WW}(p^2)} \Big|_{p \approx 0}$$

$$= \frac{e^2}{8s^2c^2M_2^2} \left( 1 - \frac{\Pi_{WW}(0)}{M_W^2} + \dots \right)$$

Now, we invert relations (order d)

$$\alpha = \hat{\alpha} \left( 1 - \frac{\Pi_{\gamma\gamma}(\hat{M}_2^2)}{\hat{M}_2^2} \right)$$

$$M_2^2 = \hat{M}_2^2 \left( 1 - \frac{\Pi_{ZZ}(\hat{M}_2^2)}{\hat{M}_2^2} \right)$$

$$s^2c^2 = \frac{\pi\alpha}{\sqrt{2}G_F\hat{M}_2^2} (1 + \Pi_R)$$

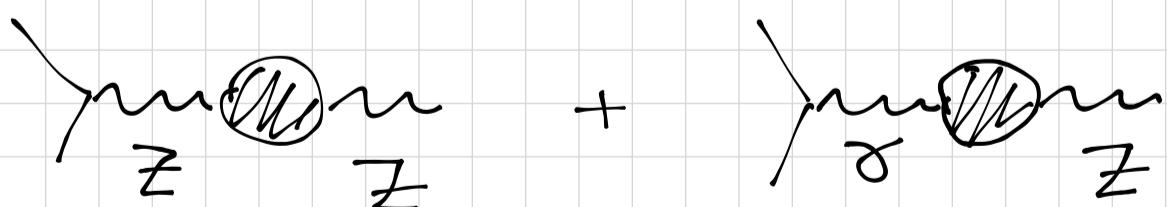
$$\text{with } \Pi_R = - \frac{\Pi_{\gamma\gamma}(\hat{M}_2^2)}{\hat{M}_2^2} + \frac{\Pi_{ZZ}(\hat{M}_2^2)}{\hat{M}_2^2} - \frac{\Pi_{WW}(0)}{M_W^2}$$

$$\hookrightarrow s^2 = \hat{s}^2 \left( 1 + \frac{\hat{c}^2}{\hat{c}^2 - \hat{s}^2} \Pi_R \right)$$

$$c^2 = \hat{c}^2 \left( 1 - \frac{\hat{s}^2}{\hat{c}^2 - \hat{s}^2} \Pi_R \right)$$

Additional complication for Z-pole asym.

$\gamma$ -Z mixing



$$\begin{aligned} \mathcal{L}_Z^{\text{eff}} = & -\frac{e}{sc} Z_\mu \left[ \left( \frac{1}{2} - s^2 \right) \bar{e}_L \gamma^\mu e_L - s^2 \bar{e}_R \gamma^\mu e_R \right] \\ & - e \frac{\Pi_{\gamma Z}(\Pi_Z^2)}{\Pi_Z^2} \left[ \bar{e}_L \gamma^\mu e_L + \bar{e}_R \gamma^\mu e_R \right] Z_\mu \end{aligned}$$

$$\Rightarrow S_{\text{eff}}^2 \equiv S^2 - \text{sc} \frac{\Pi_{\gamma Z}(M_Z^2)}{M_Z^2}$$

In terms of which

$$A_c = \frac{(\frac{1}{2} - S_{\text{eff}}^2)^2 - S_{\text{eff}}^4}{(\frac{1}{2} - S_{\text{eff}}^2)^2 + S_{\text{eff}}^4}$$


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$$M_{W, \text{pole}}^2 = C^2 M_Z^2 + \Pi_{WW}(M_W^2)$$

$$= \hat{C}^2 \hat{M}_Z^2 \left( 1 - \frac{\hat{S}^2}{\hat{C}^2 - \hat{S}^2} \Pi_R - \frac{\Pi_{ZZ}(M_Z^2)}{M_Z^2} + \frac{\Pi_{WW}^2(M_W^2)}{\hat{C}^2 M_Z^2} \right)$$

$$S_{\text{eff}}^2 = \hat{S}^2 + \frac{\hat{S}^2 \hat{C}^2}{\hat{C}^2 - \hat{S}^2} \Pi_R - \hat{S} \hat{C} \frac{\Pi_{\gamma Z}(M_Z^2)}{\hat{M}_Z^2}$$

Both: observables not used as input parameters  $\rightarrow$  consistency check

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Next: compute various  $\Pi_{ij}$

Fermionic VP  $\rightarrow$  account for  $t, b$  only

Divergent parts

$$\frac{\Pi_{\gamma\gamma}(M_Z^2)}{M_Z^2} = - \frac{2d}{\pi} \frac{1}{\epsilon} (Q_t^2 + Q_b^2) \quad d = 4 - 2\epsilon$$

$$Q_t = \frac{2}{3}$$

$$Q_b = -\frac{1}{3}$$

$$\frac{\Pi_{WW}(M_W^2)}{M_W^2} = |\mathcal{V}_{tb}|^2 \frac{3d}{4\pi S^2} \frac{1}{\epsilon} \left( \frac{m_b^2 + m_t^2}{M_W^2} - \frac{2}{3} \right)$$

$$\frac{\Pi_{Z2}(M_2^2)}{M_2^2} = \frac{\alpha}{2\pi s c} \frac{1}{\epsilon} (Q_b - Q_t + 4s^2(Q_b^2 + Q_t^2))$$

$$\frac{\Pi_{Z2}(M_2^2)}{M_2^2} = \frac{\alpha}{4\pi s^2 c^2} \frac{1}{\epsilon} \left( 3 \frac{m_b^2 + m_t^2}{M_2^2} - 2[1 + 2(Q_b - Q_t)s^2 + 4s^4(Q_b^2 + Q_t^2)] \right)$$

Important check :  $M_{W,\text{pole}}^2$  and  $S_{\text{eff}}^2$  are finite if  $|V_{tb}|^2 = 1$   
 This is not quite true, but we neglected other generations  
 If included  $\rightarrow |V_{tb}|^2 + |V_{ts}|^2 + |V_{td}|^2 = 1$

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## Low-energy EW observables

Since  $Q^2 \ll M_2^2$  it is convenient to define effective 4-fermion interactions:  
 E.g., NC processes

$$\mathcal{L}_{\nu e} = -\frac{G_F}{\sqrt{2}} \bar{\nu} \gamma_\mu (1-\gamma_5) \nu \bar{e} \gamma^\mu (g_{LV}^{ve} - g_{LA}^{ve} \gamma_5) e$$

$$\mathcal{L}_{q_L} = -\frac{G_F}{\sqrt{2}} \bar{q} \gamma_\mu (1-\gamma_5) q \sum_q [g_{LU}^{vq} \bar{q} \gamma^\mu (1-\gamma_5) q + g_{LR}^{vq} \bar{q} \gamma^\mu (1+\gamma_5) q]$$

$$\mathcal{L}_{ce} = -\frac{G_F}{\sqrt{2}} g_{AV}^{ec} \bar{e} \gamma^\mu \gamma_5 e \bar{e} \gamma^\mu e$$

$$\mathcal{L}_{eh} = -\frac{G_F}{\sqrt{2}} \sum_q [g_{AV}^{eq} \bar{e} \gamma^\mu \gamma_5 e \bar{q} \gamma_\mu q + g_{VA}^{eq} \bar{e} \gamma^\mu e \bar{q} \gamma_\mu \gamma_5 q]$$

SM prediction for  $g_{AV, VA, LA, LV}^{ij}$ :

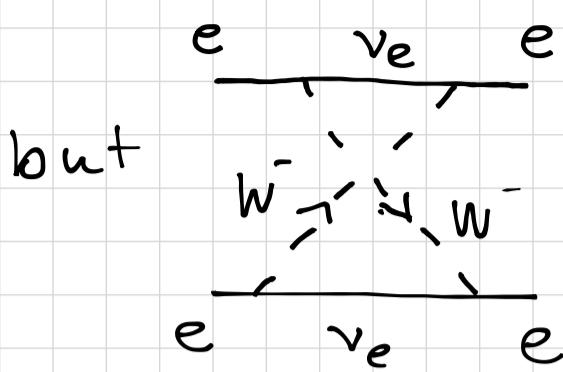
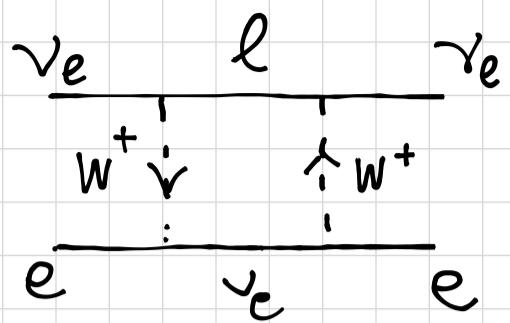
universal part (running or oblique)

+ non-universal (vertex corrections, boxes)

	SM tree	SM tot
$g_{LV}^{\nu_\mu e}$	$-\frac{1}{2} + 2\hat{S}_0^2$	-0.0398
$g_{LA}^{\nu_\mu e}$	$-\frac{1}{2}$	-0.5064
$g_{LL}^{\nu_\mu u}$	$\frac{1}{2} - \frac{2}{3}\hat{S}_0^2$	0.3458
$g_{LR}^{\nu_\mu u}$	$-\frac{2}{3}\hat{S}_0^2$	-0.1552
$g_{LU}^{\nu_\mu d}$	$-\frac{1}{2} + \frac{1}{3}\hat{S}_0^2$	-0.4288
$g_{LR}^{\nu_\mu d}$	$+\frac{1}{3}\hat{S}_0^2$	0.0777
$g_{AV}^{ec}$	$\frac{1}{2} - 2\hat{S}_0^2$	0.0227
$g_{AV}^{eu}$	$-\frac{1}{2} + \frac{4}{3}\hat{S}_0^2$	-0.1888
$g_{AV}^{ed}$	$\frac{1}{2} - \frac{2}{3}\hat{S}_0^2$	0.3419
$g_{VA}^{eu}$	$-\frac{1}{2} + 2\hat{S}_0^2$	-0.0352
$g_{VA}^{ed}$	$\frac{1}{2} - 2\hat{S}_0^2$	0.0249

Non-universal corrections numerically important

E.g. WW-box



Contributes with opposite sign

weak charge of the proton

$$Q_W^P = -2 \left( 2 \Omega_{FAV}^{eu} + g_{AV}^{ed} \right)^{\text{tree}} = 1 - 4 S_0^2$$

$$S_0^2 = 0.23857(5) \rightarrow 0.04572(20)$$

But including

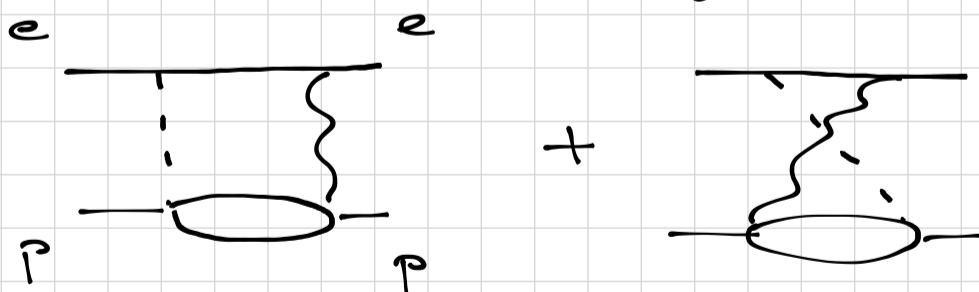
$\sim 26\%$

and

$\sim 7\%$

$$(Q_W^P)^{\text{1-loop}} = 0.0712(5)$$

Additional complication: ZZ-box strongly depends on energy



Hadronic tensor  $W^{\mu\nu} \sim \underbrace{-g^{\mu\nu} F_1^{\gamma Z} + \frac{p^\mu p^\nu}{pq} F_2^{\gamma Z}}_{V \times V} + i \epsilon^{\mu\nu\alpha\beta} \frac{p_\alpha q_\beta}{2pq} \frac{F_3^{\gamma Z}}{V \times A}$

Enough to evaluate at  $t=0$   
(exact forward limit in which the weak  
charge is defined), but keep track of  
the electron energy  $E$

For  $E=0$  only  $F_3^{\delta Z}$  contribution survives

(C-parity : weak charge  $\sim V$  :  $C = -1$

$$V \times V : (C = -1)^2 \rightarrow C = +1$$

$$V \times A : (C = -1) \times (C = +1) \rightarrow C = -1$$

CP-parity:  $\Rightarrow V \times V : P = -1, \square_{\delta Z}^V(-E) = -\square_{\delta Z}^V(E)$

$$V \times A : P = +1, \square_{\delta Z}^A(-E) = +\square_{\delta Z}^A(E)$$

For  $E=0$  exact cancellation for  $\square_{\delta Z}^V$   
between direct and crossed box;

For  $E \neq 0$  this cancellation is partial

In particular: Qweak Experiment @ JLAB  
operated with  $E = 1.165$  GeV.

It was found  $\square_{\delta Z}^V(E = 1.165 \text{ GeV}) = 0.0054(20)$

( $\sim (8 \pm 3)\%$  of  $Q_W^P$ !).

This energy dependence (and uncertainty)  
motivated the local PVES program at  
MESX:  $E = 155$  MeV for which

$$\square_{\delta Z}^V(155 \text{ MeV}) = 0.0011(3)$$

$(1.5 \pm 0.4)\%$

Look for deviations from SM predictions

Oblique parameters: modification of VP due to unknown virtual particles

$\rho$ - parameter

$$\rho = \frac{\pi_W^2}{\pi_Z^2 \cos^2 \theta_W} \quad \text{tree level}$$

$$\rho_0 = 1$$

$\rho_0 = 1$  is protected by the custodial  $SU(2)$  or isospin symmetry which is only broken by quark mass

$$\Delta \rho^t = \frac{3d}{16\pi S^2 C^2} \frac{m_t^2}{\pi_Z^2} \approx 0.008$$

Absorb all SM particle corrections in the definition  $\rightarrow$  enforce  $\rho = 1$  in SM

Deviation of  $\rho$  from 1 = Physics beyond SM

E.g. a non-degenerate fermion doublet will contribute

$$\rho - 1 = \frac{6F}{8\sqrt{2}\pi^2} \sum_n C_n \Delta m_n^2, \quad C_n = N_c \begin{cases} 1, & \text{for } \ell \\ 3, & \text{for } q \end{cases}$$

$$\Delta m_n^2 = m_{n1}^2 + m_{n2}^2 - \frac{4m_{n1}^2 m_{n2}^2}{m_{n1}^2 - m_{n2}^2} \ln \frac{m_{n1}}{m_{n2}}$$

$$\rho = 1.00038(20) \quad \underbrace{\left(14 \text{ GeV}\right)^2}_{\rightarrow} \leq \sum_n \frac{C_n}{3} \Delta m_n^2 \leq \left(49 \text{ GeV}\right)^2$$

Negative contributions to  $\rho$  from  
extra Higgs doublets or Majorana  $\nu$

In general, Peskin - Takeuchi parameters:

$$\hat{\alpha}(M_Z) T \equiv \frac{\Pi_{WW}^{\text{new}}(0)}{M_W^2} - \frac{\Pi_{ZZ}^{\text{new}}(0)}{M_Z^2}$$

$S, T, U$

$$\frac{\hat{\alpha}(M_Z)}{4\hat{S}_Z^2\hat{C}_Z^2} S = \frac{\Pi_{ZZ}^{\text{new}}(M_Z^2) - \Pi_{ZZ}^{\text{new}}(0)}{M_Z^2}$$

$$\frac{\hat{\alpha}(M_Z)}{4\hat{S}_Z^2} (S+U) = \frac{\Pi_{WW}^{\text{new}}(M_W^2) - \Pi_{WW}^{\text{new}}(0)}{M_W^2}$$

$$\Pi_{ij}^{\text{new}} = \Pi_{ij} - \Pi_{ij}^{\text{SM}}$$

$$T = \frac{\rho - 1}{\alpha}$$

For a degenerate heavy chiral multiplet

$$S = \frac{c}{3\pi} \sum_r \left[ t_{rL}^3 - t_{rR}^3 \right]^2$$

$$U \approx 0$$

Global fit

$$S = 0.00 \pm 0.07$$

$$T = 0.05 \pm 0.06$$

A multiparameter fit is consistent with

$$\rho = 1$$

Instead,  $\rho = 1.00038(20)$  was obtained

enforcing  $S=T=U=0 \rightarrow$  hints on BSM

If fixing  $T=U=0$   $S$  can be used to constrain new fermion generations

Global fits exclude new SM fermions at the 86 level.

$S, T, U$  formalism does not include new physics coupling directly to SM fermions.

For instance, heavy  $Z'$ , leptoquark, technicolor etc. can be adequately described by EFT with higher-dimensional op's

$$\mathcal{L} = \mathcal{L}_{SM} + \sum_{d>4} \sum_i \frac{c_i}{x^{d-4}} O_i \quad \Lambda \gg v$$

In this language, 4-fermion op's start at  $d=6$ , and  $S, T$  can be identified with 2 combinations of  $c_i$ .  $U \neq 0$  only appears at  $d=8$

For running WMA

