

Lecture 15

Electroweak unification

1. What (and why!) needs to be unified?

Electromagnetism

$$\mathcal{L}_{\text{QED}} = -\frac{1}{4}(\bar{F}_{\mu\nu})^2 + \bar{\psi}(i\gamma^\mu - m)\psi$$

Abelian gauge-field theory

renormalizable at each order

Mathematically well-defined at low energy

(High-precision predictions for low-energy uncertainties: atomic spectra, $(g-2)_{\mu,e}$, ...)

However: running $\alpha(\mu) = \frac{\alpha_0}{1 - \frac{\alpha_0}{3\pi} \ln \frac{\mu^2}{\lambda_{\text{QED}}^2}}$

has a Landau pole for $\mu \gg$

$$\mu \sim m_e e^{3\pi/2\alpha_0} \sim 10^{286} \text{ eV}$$

As a result, QED is undefined in the UV

Means, QED cannot be a complete theory.

[This is the case for any Abelian QFT!]

Weak interaction

a. what do we understand under w.i.?

In primis: β -decay

$$\text{Muon} \quad \mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$$

$$\text{Neutron} \quad n \rightarrow p + e^- + \bar{\nu}_e$$

$$\text{Meson} \quad \pi, K \rightarrow l, \nu_e ; \quad \pi^+ \rightarrow \pi^0 + e^+ + \nu_e$$

$$\text{Nuclear} \quad A_i \rightarrow A_f + e^\pm + \nu_e$$

Effectively described by a 4-Fermi theory

$$\mathcal{L}_{\mu \rightarrow e\nu} = - \frac{G_F}{\sqrt{2}} \bar{u}_{\nu_\mu} \gamma^\mu (1-\gamma_5) u_\mu \bar{u}_e \gamma_\mu (1-\gamma_5) u_{\nu_e}$$

$$\mathcal{L}_{n \rightarrow p e \bar{\nu}} = - \frac{G_F}{\sqrt{2}} \bar{u}_p \gamma^\mu (1-\gamma_5) u_n \bar{e} \gamma_\mu (1-\gamma_5) u_{\nu_e}$$

Note: original Fermi contact interaction looked like $\sim G_F \bar{u}_p u_n \bar{e} v_n$, before parity violation was discovered

1956: Lee, Yang wrote down a general effective 4-fermion $n p e \bar{\nu}$ Lagr. with S, V, T, PS, A, PT interactions

1957: We discovered that leptons originating from β decay are left-handed \rightarrow hence the L projector $\frac{1}{2}(1-\gamma_5) = P_L$
 $(P_L^2 = P_L, P_L P_R = 0)$

Originally, no way to distinguish whether β -decay is V-A combination or S-PS: need to explain $n \rightarrow p e \bar{\nu}$; $\pi \rightarrow \mu \bar{\nu}$; $0^+ \rightarrow 0^+ e \bar{\nu}$ decays and similar

If using the effective L. at tree-level
 \rightarrow can roughly describe all decays

How do quantum corrections modify the coupling $G_F \sim 1.166 \cdot 10^{-5} \text{ GeV}^{-2}$?

Here's the problem: 4F interaction has $D=6$ ($[G_F] = -2$) and is non-renormalizable.
 How can the physical coupling be extracted if quantum corrections $\sim G_F \Lambda^2$, Λ -UV cutoff will strongly modify $G_F^{\text{eff.}}$ at each order?

By that time muon lifetime τ_μ was very precisely measured

$$\tau_\mu = 2.1969811(22) \mu\text{s}$$

$\tau_\mu \leftrightarrow G_F$ at tree-level:

$$\frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^3} F(x) \quad \text{phase-space factor}$$

$$F(x) = 1 - 8x - 12x^2 \ln x + 8x^3 - x^4, \quad x = \frac{m_e}{M_\mu}$$

How is F - this result modified in presence of RC?

$$\frac{1}{T_F} = \frac{G_F^2 m_\mu^5}{192\pi^3} F(x) [1 + \delta_\mu]$$

It turned out, RC in V-A theory were finite, but UV-divergent in S-PS.

At present: known at 2-loop:

$$\delta_\mu = \frac{\alpha}{2\pi} \left(\frac{25}{4} - \tau_1^2 \right) \left[1 + \frac{2\alpha}{3\tau_1} \ln \frac{1}{x} \right] + 6.700 \left(\frac{\alpha}{\pi} \right)^2 + \dots$$

$$= -4.19818 \cdot 10^{-3}$$

$$\Rightarrow G_F = 1.1663788(7) \cdot 10^{-5} \text{ GeV}^{-2}$$

One of the most precisely known physical constants

Does it work for neutron decay?
Surprisingly, no!

τ_n is UV-divergent ($\ln \lambda$)
in both S-PS and V-A theories

This was the situation in mid-1960's

Non-renormalizable theory of β -decay
Renormalizable QED with severe problems
at high energies

Solution: assume that weak interaction is conveyed by massive vector

bosons,

$$m_W \sim \frac{1}{\sqrt{G_F/\sqrt{2}}} \sim 246 \text{ GeV}$$

Then, the guess is that there should be the $SU(2)$ sector (responsible for weak interaction: acts on doublets $(\bar{n})(\bar{e})(\bar{u})(\bar{d})$)

and $U(1)$ sector (responsible for QED)

The two sectors have to be connected somehow to cure each other's problems!

Glashow - Weinberg - Salam electroweak model

$SU(2)$: non-Abelian gauge theory,
associated field W_μ^a

$U(1)$: Abelian gauge theory,
associated field $B_\mu \rightarrow$ hypercharge

Spontaneous symmetry breaking to generate weak boson mass

→ Higgs boson doublet (H^+, H^0)

Gauge + Higgs sector:

$$\mathcal{L} = -\frac{1}{4}(W_{\mu\nu}^a)^2 - \frac{1}{4}(B_{\mu\nu})^2 + (\bar{D}_\mu H)^+ (D^\mu H) + m^2 H^+ H - \lambda (H^+ H)^2$$

All details are already familiar

Covariant derivative:

$$D_\mu H = \partial_\mu H - ig W_\mu^\alpha T^\alpha H - \frac{1}{2}ig' B_\mu H$$

$$\begin{pmatrix} g \\ g' \end{pmatrix} \rightarrow \begin{matrix} \text{SU(2)} \\ \text{U(1)}_Y \end{matrix} \quad \text{couplings}$$

Higgs hypercharge $Y_H = \frac{1}{2}$

Higgs potential $V(H) = -m^2(H^+H) + \lambda(H^+H)^2$
develops a VEV

Linear G-model:

$$H = \exp\left[2i \frac{\pi^\alpha \tau^\alpha}{v}\right] \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix}$$

$h \rightarrow$ real scalar (G-meson)

Unitary gauge \rightarrow shift $\tau^\alpha \rightarrow 0$

Start with integrating out h (non-linear G)

$$H \rightarrow \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Kinetic term becomes [recall $\vec{\tau} = \frac{1}{2} \vec{G}$]

$$|D_\mu H|^2 = g^2 \frac{v^2}{g'} (01) \begin{bmatrix} W_\mu^3 + \frac{g'}{g} B_\mu & W_\mu^1 - i W_\mu^2 \\ W_\mu^1 + i W_\mu^2 & -W_\mu^3 + \frac{g'}{g} B_\mu \end{bmatrix} \times$$

{ the same because T^α hermitian } $\begin{bmatrix} \underline{\quad} & \underline{\quad} & \underline{\quad} \end{bmatrix} (01)$

$$= g^2 \frac{v^2}{8} \left[(W_\mu^1)^2 + (W_\mu^2)^2 + \left(\frac{g'}{g} B_\mu - W_\mu^3 \right)^2 \right].$$

As usual: break $SU(2)$ symmetry, integrate out ϕ , go to unitary gauge
 \rightarrow land at a Proca Lagrangian for massive vector boson

$$m_A^2 = g^2 \frac{v^2}{4}$$

Except now we also have an unbroken $U(1)$
 \Rightarrow out of 4 k^+ soft modes ($3 = 2-1$ $SU(2)$
 $+ 1$ $U(1)$)

We only generated 3 mass terms

Also, upon manipulating the Higgs doublet

$$\begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

the initial gauge fields mixed

Define weak mixing angle Θ_W :

$$\tan \Theta_W = \frac{g'}{g}$$

$$\Rightarrow \begin{pmatrix} Z^\mu \\ A^\mu \end{pmatrix} = \begin{pmatrix} \cos \Theta_W & -\sin \Theta_W \\ \sin \Theta_W & \cos \Theta_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}$$

Physical fields

Simple rotation matrix

$\det |..| = 1$

Initial fields



NC = neutral current

$$\mathcal{L}_{\text{kin}}^{\text{NC}} = -\frac{1}{4}(F_{\mu\nu})^2 - \frac{1}{4}(Z_{\mu\nu})^2 + \frac{1}{2}m_Z^2 Z_\mu^2$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$\mathcal{L}_{\text{kin}}^{\text{CC}} = -\frac{1}{2}W_{\mu\nu}^+ W^{\mu\nu,-} + m_W^2 W_\mu^+ W^\mu_-$$

CC = charged current

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp i W_\mu^2)$$

Additionally: a series of gauge boson interactions → Exercise following Schwartz 29.1

Here a few notations were introduced

$$m_W = \frac{g v}{2}$$

$$m_Z = \frac{v}{2} \sqrt{g^2 + g'^2} = \frac{g v}{2 \cos \theta_W} = \frac{m_W}{\cos \theta_W}$$

$$\text{QED coupling } e = g \sin \theta_W = g' \cos \theta_W$$

$$\cos \theta_W = \frac{m_W}{m_Z} = \frac{g}{\sqrt{g^2 + g'^2}} \quad (\text{at tree level} \rightarrow \text{equiv.})$$

Fermion sector

Within each generation:

$$l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \quad l = e, \mu, \tau$$

SU(2) only couples to L doublets!

$$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad q = \begin{matrix} u^c \\ d^c \\ s^c \end{matrix}$$

Right-handed singlets: e_R, ν_R, u_R, d_R

$$\begin{aligned} \mathcal{L}_f = & i \bar{\ell}_L (\not{p} - ig W^a \tau^a - ig' Y_L \not{B}) \ell_L \\ & + i \bar{e}_R (\not{p} - ig' Y_e \not{B}) e_R + i \bar{\nu}_R (\not{p} - ig' Y_\nu \not{B}) \nu_R \\ & + i \bar{q}_L (\not{p} - ig W^a \tau^a - ig' Y_Q \not{B}) q_L \\ & + i \bar{u}_R (\not{p} - ig' Y_u \not{B}) u_R + i \bar{d}_R (\not{p} - ig' Y_d \not{B}) d_R \end{aligned}$$

What are the charges

$$W^a \tau^a = W^+ \tau^- + W^- \tau^+ + W^3 \tau^3$$

Charge current:

$$\begin{aligned} \mathcal{L}_f^{CC} = & g \bar{\ell}_L (W^+ \tau^- + W^- \tau^+) \ell_L \\ & + g \bar{q}_L (W^+ \tau^- + W^- \tau^+) q_L \end{aligned}$$

The same strength in quark and lepton sect.

Neutral current:

$$\mathcal{L}_f^{NC} = \left(\begin{array}{c} \bar{\nu}_L \\ e_L \end{array} \right) \left(\frac{g}{2} W^3 \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + g' Y_L \not{B} \mathbb{1} \right) \left(\begin{array}{c} \nu_L \\ e_L \end{array} \right)$$

$$+ g' Y_e \bar{e}_R \not{B} e_R + g' Y_\nu \bar{\nu}_R \not{B} \nu_R$$

$$\begin{pmatrix} W^3 \mu \\ B^\mu \end{pmatrix} = \begin{pmatrix} \cos \theta_W & \sin \theta_W \\ -\sin \theta_W & \cos \theta_W \end{pmatrix} \begin{pmatrix} Z \\ A \end{pmatrix}$$

1. Electromagnetic part

$$g \sin \theta_W \equiv e = g' \cos \theta_W$$

$$e \left[\bar{\nu}_L \left(\frac{1}{2} + Y_L \right) \not{A} \nu_L + Y_\nu \bar{\nu}_R \not{A} \nu_R \right. \\ \left. + \bar{e}_L \left(-\frac{1}{2} + Y_L \right) \not{A} e_L + Y_e \bar{e}_R \not{A} e_R \right]$$

E.m. interaction is the same for L and R !

$$\Rightarrow -\frac{1}{2} + Y_L = Y_e = -1 \Rightarrow Y_L = -\frac{1}{2}$$

$$\frac{1}{2} + Y_L = Y_\nu = 0 \quad Y_e = -1 \\ \quad \quad \quad Y_\nu = 0$$

Similarly, for quarks

$$e \left[\bar{u}_L \left(\frac{1}{2} + Y_Q \right) \not{A} u_L + Y_u \bar{u}_R \not{A} u_R \right] \\ + e \left[\bar{d}_L \left(-\frac{1}{2} + Y_Q \right) \not{A} d_L + Y_d \bar{d}_R \not{A} d_R \right]$$

$$\frac{2}{3} = \frac{1}{2} + Y_Q = Y_u \quad Y_Q = \frac{1}{6} \\ -\frac{1}{3} = -\frac{1}{2} + Y_Q = Y_d \quad Y_u = \frac{2}{3} \\ \quad \quad \quad Y_d = -\frac{1}{3}$$

Completely analogous for Z

$$\bar{e}_L \left(g \cos \theta_W \bar{T}^3 - g' \sin \theta_W Y_L \not{1} \right) \not{A} e_L$$

$$-g' \sin \theta_W Y_e \bar{e}_R \not{A} e_R - g' \sin \theta_W Y_\nu \bar{\nu}_R \not{A} \nu_R$$

$$\text{with } \bar{T}^3 = \frac{1}{2} T^3$$

1. Since $\gamma_5 = 0$, γ_R completely decouples!

The only way it can contribute:

via Dirac mass term \rightarrow come back later

2. Decomposing

$$\mathcal{L} = \dots + \frac{e}{\sin \theta_W} Z_\mu J_Z^\mu + e A_\mu J_{EM}^\mu$$

$$\text{with } J_{EM}^\mu = J^{\mu 3} + J_Y^\mu$$

$$= \underbrace{\bar{\psi}^L \gamma^\mu \gamma^3 \psi^L}_{J^{\mu 3}} + \underbrace{\gamma^L \bar{\psi}^L \gamma^\mu \psi^L + \gamma^R \bar{\psi}^R \gamma^\mu \psi^R}_{J_Y^\mu}$$

This way of writing is generalized to $\psi^L = \psi_L, q_L$
and $\psi^R = e^R, u^R, d^R (\nu^R)$

$$\Rightarrow J_Z^\mu = \frac{1}{\cos \theta_W} [J^{\mu 3} - \sin^2 \theta_W J_{EM}^\mu]$$

Coupling of Z -boson:

$$\mathcal{L} = \frac{e}{\sin \theta_W \cos \theta_W} Z^\mu [J_\mu^3 - \sin^2 \theta_W J_{EM}^\mu]$$

Define weak charge of a fermion:

$$Q_W^f = T_f^3 - 2 \sin^2 \theta_W Q_f$$

T_f^3 - weak isospin: $+ \frac{1}{2}$ for ν_L, u_L
 $- \frac{1}{2}$ for e_L, d_L
 0 for l, ll, R

$$Q_f = T_f^3 + Y_f$$

$$\text{Conventionally: } Q_W^f = 2(\bar{t}^3 - 2\sin^2 \theta_W Q^f)$$

$$Q_W^e = -1 + 4 \sin^2 \theta_W \quad Q_W^\nu = 1$$

$$Q_W^u = 1 - \frac{8}{3} \sin^2 \theta_W \quad Q_W^d = -1 + \frac{4}{3} \sin^2 \theta_W$$

$$Q_W^P = 2Q_W^u + Q_W^d = 1 - 4 \sin^2 \theta_W \quad Q_W^n = Q_W^u + 2Q_W^d = -1$$

Some numbers and facts

EW model predictions: $m_Z = \frac{m_W}{\cos \theta_W} > m_W$

weak interaction universal: $g_e = g_q$

γ_R is completely sterile to EW interaction

With measured $m_Z \approx 91.2 \text{ GeV}$

$m_W \approx 80.4 \text{ GeV}$

$\Rightarrow \sin^2 \theta_W \approx 0.223 \rightarrow \text{close to } 1/4$

$\Rightarrow Q_W^P = 1 - 4 \bar{s}_W^2 \ll |Q_W^n| = 1$

$|Q_W^e| = 1 - 4 \bar{s}_W^2 \ll |Q_W^\nu| = 1$

MESA: future exp. program to measure
 Q_W^P to better than 2%

$$m_W = \frac{g_0}{2} \Rightarrow v = \frac{2m_W \sin \theta_W}{e} \approx 250 \text{ GeV}$$

$$e \approx 0.303$$